Liceo "Carducci" Volterra - Classi 1A, 1B Scientifico - Francesco Daddi - 19 marzo 2009

Esercizi svolti sulle frazioni algebriche

Esercizio 1.

$$\frac{x^2-4\,x+3}{x-1}+\frac{2-x}{x^2-4}=\frac{(x-1)(x-3)}{x-1}-\frac{x-2}{(x-2)(x+2)}=x-3-\frac{1}{x+2}=$$

$$=\frac{(x-3)(x+2)-1}{x+2}=\frac{x^2-x-7}{x+2}\,.$$

Esercizio 2.

$$\frac{x^2+2\,x+1}{1-x^2}-\frac{x^3-1}{x-1}+\frac{2-8\,x^2}{4\,x^2-1}=\frac{(x+1)^2}{(1+x)(1-x)}-\frac{(x-1)(x^2+x+1)}{x-1}+\frac{2(1-4\,x^2)}{(4\,x^2-1)}=$$

$$=\frac{x+1}{1-x}-(x^2+x+1)-2=\frac{x+1-(x^2+x+1)(1-x)-2(1-x)}{1-x}=\frac{x^3+3\,x-2}{1-x}\,.$$

Esercizio 3.

$$\frac{-2\,x^2+10\,x-12}{x^2-6\,x+9}-\frac{1-x}{x^2-1}+\frac{3\,x-x^2-2}{x^3-2\,x^2-5\,x+6}=$$

$$=\frac{-2(x-2)(x-3)}{(x-3)^2}+\frac{x-1}{(x-1)(x+1)}-\frac{(x-1)(x-2)}{(x-1)(x+2)(x-3)}=\frac{-2(x-2)}{(x-3)}+\frac{1}{x+1}-\frac{x-2}{(x+2)(x-3)}=$$

$$=\frac{-2(x-2)(x+1)(x+2)+(x-3)(x+2)-(x-2)(x+1)}{(x-3)(x+1)(x+2)}=\frac{-2\,x^3-2\,x^2+8\,x+4}{(x-3)(x+1)(x+2)}\,.$$

Esercizio 4.

$$\frac{1-x}{(x-1)^2}-\frac{x^3+1}{(x+1)^2}+\frac{3\,x^2-4\,x+1}{1-x^2}=-\frac{x-1}{(x-1)^2}-\frac{(x+1)(x^2-x+1)}{(x+1)^2}+\frac{(x-1)(3\,x-1)}{(1-x)(1+x)}=$$

$$=-\frac{1}{x-1}-\frac{x^2-x+1}{x+1}-\frac{(x-1)(3\,x-1)}{(x-1)(x+1)}=-\frac{1}{x-1}-\frac{x^2-x+1}{x+1}-\frac{3\,x-1}{x+1}=$$

$$=\frac{-(x+1)-(x^2-x+1)(x-1)-(3\,x-1)(x-1)}{(x-1)(x+1)}=\frac{-x^3-x^2+x-1}{x^2-1}\,.$$

Esercizio 5.

$$\frac{x^2}{2}-\frac{(1-x)^2}{x^3-x}-\frac{2}{1-x}+(x-3)\,\frac{2\,x-x^2-1}{(1-x^2)^2}=\frac{x^2}{2}-\frac{(x-1)^2}{x(x-1)(x+1)}+\frac{2}{x-1}-(x-3)\,\frac{(x-1)^2}{(x^2-1)^2}=$$

$$=\frac{x^2}{2}-\frac{x-1}{x(x+1)}+\frac{2}{x-1}-\frac{(x-3)(x-1)^2}{(x-1)^2(x+1)^2}=\frac{x^2}{2}-\frac{x-1}{x(x+1)}+\frac{2}{x-1}-\frac{x-3}{(x+1)^2}=$$

$$=\frac{x^2x(x-1)(x+1)^2-(x-1)2(x+1)(x-1)+2\cdot 2x(x+1)^2-(x-3)2\,x(x-1)}{2\,x(x-1)(x+1)^2}=$$

$$=\frac{x^6+x^5-x^4-x^3+18\,x^2-2}{2\,x(x-1)(x+1)^2}\,.$$

Liceo "Carducci" Volterra - Classi 1A, 1B Scientifico - Francesco Daddi - 7 marzo 2009

Esercizi sulle frazioni algebriche

Svolgi le seguenti espressioni:

Esercizio 1. $\dfrac{x^2-3\,x}{x^2-1}-\dfrac{2}{x-1}+\dfrac{x-4}{x+1}$

Esercizio 2. $\dfrac{x}{x^3-x}+\dfrac{-2+4\,x}{x^2-1}-\dfrac{2\,x-x^2}{x+1}$

Esercizio 3. $\dfrac{2\,x-26}{x^2-4\,x+3}-\dfrac{3\,x+5}{x^2-1}+\dfrac{x^2+1}{x-3}+\dfrac{3\,x-2}{x+1}$

Esercizio 4. $\dfrac{x-3}{x^2+2\,x+1}+\dfrac{x^2}{x^2+5\,x+4}$

Esercizio 5. $\dfrac{x-1}{x^3-4\,x^2+x-4}-\dfrac{1}{x-4}+\dfrac{x^2-1}{x^2-2\,x-8}$

Esercizio 6. $\dfrac{x-1}{x^3-4\,x^2+x-4}-\dfrac{1}{x-4}+\dfrac{x^2-1}{x^2-2\,x-8}$

Esercizio 7. $\dfrac{2\,x-1}{x-1}+\dfrac{x-2}{(x-1)^2}-\dfrac{x^2-4}{(x-1)^3}+\dfrac{x+1}{(x-1)^4}$

Esercizio 8. $\dfrac{2\,x}{x-2}+\dfrac{(x-1)(x-18)}{x^2-4}-\dfrac{x+1}{(x+2)^2}$

Esercizio 9. $\dfrac{x^3-2\,x}{x^3-2\,x^2-5\,x+6}+\dfrac{3\,x-6}{x^3-3\,x^2-6\,x+8}$

Esercizio 10. $\dfrac{(x-y)^3-1}{x-y}+\dfrac{4\,x-4\,y}{x^2-y^2}$

Esercizio 11. $\dfrac{x-y}{x^3-y^3}-\dfrac{x+y}{x^2-y^2}$

Esercizio 12. $\dfrac{(x^2+1)(x-1)}{(x^2-3\,x+2)^2}+\dfrac{x+3}{(x-1)^2}+\dfrac{x}{x-2}+\dfrac{1}{5\,x-2-4\,x^2+x^3}-\dfrac{x+1}{13\,x^2-12\,x+4-6\,x^3+x^4}$

Soluzioni esercizi sulle frazioni algebriche 7 marzo 2009

```
> (x^2-3*x)/(x^2-1)-(2)/(x-1)+(x-4)/(x+1);
```

$$\frac{x^2-3x}{-1+x^2}-\frac{2}{-1+x}+\frac{x-4}{1+x}$$

```
> simplify(%);
```

$$\frac{2\,(x^2-5x+1)}{-1+x^2}$$

```
> x/(x^3-x)+(-2+4*x)/(x^2-1)-(2*x-x^2)/(x+1);
```

$$\frac{x}{-x+x^3}+\frac{-2+4x}{-1+x^2}-\frac{2x-x^2}{1+x}$$

```
> simplify(%);
```

$$\frac{x^3-3x^2+6x-1}{-1+x^2}$$

```
> (2*x-26)/(x^2-4*x+3)-(3*x+5)/(x^2-1)+(x^2+1)/(x-3)+(3*x-2)/(x+1);
```

$$\frac{2x-26}{-4x+x^2+3}-\frac{3x+5}{-1+x^2}+\frac{x^2+1}{x-3}+\frac{3x-2}{1+x}$$

```
> simplify(%);
```

$$\frac{x^3+6x^2+3x+6}{-1+x^2}$$

```
> (x-3)/(x^2+2*x+1)+(x^2)/(x^2+5*x+4);
```

$$\frac{x-3}{x^2+2x+1}+\frac{x^2}{x^2+5x+4}$$

```
> simplify(%);
```

$$\frac{x^3+2x^2+x-12}{(x^2+5x+4)\,(1+x)}$$

```
> (x-1)/(x^3-4*x^2+x-4)-(1)/(x-4)+(x^2-1)/(x^2-2*x-8);
```

$$\frac{-1+x}{x^3-4x^2+x-4}-\frac{1}{x-4}+\frac{-1+x^2}{x^2-2x-8}$$

```
> simplify(%);
```

$$\frac{x^4-x^3-x^2-5}{(x^2-2x-8)\,(x^2+1)}$$

```
> (x-1)/(x^2-5*x)-(2-x)/(x^2-10*x+25)+(1)/(x^2-x)+(2*x)/(x^2-6*x+5);
```

$$\frac{-1+x}{x^2-5x}-\frac{2-x}{x^2-10x+25}+\frac{1}{x^2-x}+\frac{2x}{x^2-6x+5}$$

```
> simplify(%);
```

$$\frac{4x^3-19x^2+3x+20}{(x^2-6x+5)\,x\,(x-5)}$$

```
> (2*x-1)/(x-1)+(x-2)/((x-1)^2)-(x^2-4)/((x-1)^3)+(x+1)/((x-1)^4);
```

$$\frac{-1+2x}{-1+x}+\frac{x-2}{(-1+x)^2}-\frac{x^2-4}{(-1+x)^3}+\frac{1+x}{(-1+x)^4}$$

```
> simplify(%);
```

$$\frac{-4+5x+6x^2-7x^3+2x^4}{(-1+x)^4}$$

```
> (2*x)/(x-2)+((x-1)*(x-18))/(x^2-4)-(x+1)/((x+2)^2);
```

$$\frac{2x}{x-2}+\frac{(-1+x)(x-18)}{x^2-4}-\frac{1+x}{(x+2)^2}$$

```
> simplify(%);
```

$$\frac{3x^2-4x-19}{(x+2)^2}$$

```
> (x^3-2*x)/(x^3-2*x^2-5*x+6)+(3*x-6)/(x^3-3*x^2-6*x+8);
```

$$\frac{-2x+x^3}{x^3-2x^2-5x+6}+\frac{3x-6}{x^3-3x^2-6x+8}$$

```
> simplify(%);
```

$$\frac{x^4-4x^3+x^2-7x+18}{(x^3-3x^2-6x+8)(x-3)}$$

```
> ((x-y)^3-1)/(x-y)+(4*x-4*y)/(x^2-y^2);
```

$$\frac{(x-y)^3-1}{x-y}+\frac{4x-4y}{x^2-y^2}$$

```
> simplify(%);
```

$$\frac{y^4-2xy^3+2yx^3+5y-x^4-3x}{-x^2+y^2}$$

```
> (x-y)/(x^3-y^3)-(x+y)/(x^2-y^2);
```

$$\frac{x-y}{x^3-y^3}-\frac{y+x}{x^2-y^2}$$

```
> simplify(%);
```

$$\frac{y^2+y+xy+x^2-x}{(-x+y)(y^2+xy+x^2)}$$

```
> ((x^2+1)*(x-1))/((x^2-3*x+2)^2)+(x+3)/((x-1)^2)+(x)/(x-2)+(1)/(5*x-2-
4*x^2+x^3)-(x+1)/(13*x^2-12*x+4-6*x^3+x^4);
```

$$\frac{(x^2+1)(-1+x)}{(x^2-3x+2)^2}+\frac{x+3}{(-1+x)^2}+\frac{x}{x-2}+\frac{1}{5x-2-4x^2+x^3}$$
$$-\frac{1+x}{13x^2-12x+4-6x^3+x^4}$$

```
> simplify(%);
```

$$\frac{x^4-2x^3+3x^2-9x+8}{13x^2-12x+4-6x^3+x^4}$$

Liceo "Carducci" Volterra - Classi 1A, 1B Scientifico - Francesco Daddi - 9 marzo 2009

Esercizi sulle frazioni algebriche (foglio 1)

Svolgi le seguenti espressioni:

Esercizio 1. $-\dfrac{1}{x+1}+\dfrac{x-2}{x^3-x}+\dfrac{1}{x}$

Esercizio 2. $\dfrac{2\,a}{2\,a^2-8}-\dfrac{2}{2\,a^2-4\,a}+\dfrac{a}{a^2+2\,a}$

Esercizio 3. $\dfrac{x}{x^2-2\,x+1}+\dfrac{x-2}{x^2+3\,x-4}-\dfrac{2}{x+4}$

Esercizio 4. $\dfrac{1}{2\,x^2-2}-\dfrac{1}{4\,x-4}+\dfrac{1}{2\,x+2}$

Esercizio 5. $\dfrac{1}{x-1}-\dfrac{2}{x^2-2\,x}+\dfrac{1}{x^2-3\,x+2}$

Esercizio 6. $\dfrac{x}{x^3-1}-\dfrac{1}{2\,x+2}-\dfrac{1}{x^2-1}+\dfrac{x^2}{2\,x^3-2}$

Esercizio 7. $\dfrac{1}{x^2-1}-\dfrac{2}{x^2+3\,x-4}+\dfrac{1}{x^2-2\,x-3}$

Esercizio 8. $\dfrac{3}{x^3-3\,x^2+4}+\dfrac{1}{x^2-x-2}$

Esercizio 9. $\dfrac{-x}{x^3+x^2y+xy^2+y^3}+\dfrac{x}{x^3-x^2y+xy^2-y^3}-\dfrac{1}{x^2-y^2}$

Esercizio 10. $\dfrac{2}{x^3-y^3}+\dfrac{2x+2y}{(x^2+y^2)^2-x^2y^2}-\dfrac{2\,xy}{(x^3-y^3)(x^2+y^2-xy)}$

Esercizio 11. $\dfrac{1}{x^3+x^2-4\,x-4}+\dfrac{1}{x^2+3\,x+2}+\dfrac{2}{x^2-x-2}$

Esercizio 12. $\dfrac{2\,a^2-4\,a+3}{a^3+1}+\dfrac{a}{a^2-a+1}-\dfrac{5}{2(a+1)}$

Esercizio 13. $\dfrac{x}{x+y-2}-\dfrac{x}{x+y+2}+\dfrac{4\,y-8}{x^2+2\,xy+y^2-4}$

Esercizio 14. $\dfrac{a}{a-2}-\dfrac{2\,a}{a+1}+\dfrac{a}{a-1}+\dfrac{5\,a^2-14}{a^3-2\,a^2-a+2}$

Esercizio 15. $\dfrac{x}{2\,x^3+3\,x^2+3\,x+1}+\dfrac{x-1}{2\,x+1}-\dfrac{2-x}{x^2-4}+\dfrac{x}{x^2+x+1}-\dfrac{1}{x+2}$

Esercizio 16. $\dfrac{1}{(x+y)^2-xy}-\dfrac{x^2+(x+y)(y+1)}{x^3-y^3}$

Soluzioni esercizi 9 marzo 2009 (foglio 1)

1] `-1/(x+1)+(x-2)/(x^3-x)+1/x;simplify(%);`

$$-\frac{1}{x+1}+\frac{x-2}{x^3-x}+\frac{1}{x}$$

$$\frac{2\,x-3}{x\,(x^2-1)}$$

2] `(2*a)/(2*a^2-8)-(2)/(2*a^2-4*a)+(a)/(a^2+2*a);simplify(%);`

$$\frac{2\,a}{2\,a^2-8}-\frac{2}{2\,a^2-4\,a}+\frac{a}{a^2+2\,a}$$

$$\frac{2\,a+1}{a\,(a+2)}$$

3] `(x)/(x^2-2*x+1)+(x-2)/(x^2+3*x-4)-(2)/(x+4);simplify(%);`

$$\frac{x}{1-2\,x+x^2}+\frac{x-2}{x^2+3\,x-4}-\frac{2}{x+4}$$

$$\frac{5\,x}{(x+4)\,(x-1)^2}$$

4] `(1)/(2*x^2-2)-(1)/(4*x-4)+1/(2*x+2);simplify(%);`

$$\frac{1}{2\,x^2-2}-\frac{1}{4\,x-4}+\frac{1}{2+2\,x}$$

$$\frac{1}{4\,(x+1)}$$

5] `1/(x-1)-2/(x^2-2*x)+1/(x^2-3*x+2);simplify(%);`

$$\frac{1}{x-1}-\frac{2}{x^2-2\,x}+\frac{1}{x^2-3\,x+2}$$

$$\frac{1}{x}$$

6] `x/(x^3-1)-1/(2*x+2)-1/(x^2-1)+x^2/(2*x^3-2);simplify(%);`

$$\frac{x}{x^3-1}-\frac{1}{2+2\,x}-\frac{1}{x^2-1}+\frac{x^2}{2\,x^3-2}$$

$$\frac{1}{2\,(x^2+x+1)}$$

7] `1/(x^2-1)-2/(x^2+3*x-4)+1/(x^2-2*x-3);simplify(%);`

$$\frac{1}{x^2-1}-\frac{2}{x^2+3\,x-4}+\frac{1}{x^2-2\,x-3}$$

$$\frac{2\,(4\,x-5)}{(x^2+3\,x-4)\,(x^2-2\,x-3)}$$

8] `3/(x^3-3*x^2+4)+1/(x^2-x-2);simplify(%);`

$$\frac{3}{x^3-3x^2+4}+\frac{1}{x^2-x-2}$$

$$\frac{1}{(x-2)^2}$$

9] `-x/(x^3+x^2*y+x*y^2+y^3)+x/(x^3-x^2*y+x*y^2-y^3)-1/(x^2-y^2);simplify(%);`

$$-\frac{x}{x^3+x^2y+xy^2+y^3}+\frac{x}{x^3-x^2y+xy^2-y^3}-\frac{1}{x^2-y^2}$$

$$\frac{-x+y}{(y+x)(y^2+x^2)}$$

10] `2/(x^3-y^3)+(2*x+2*y)/((x^2+y^2)^2-x^2*y^2)-2*x*y/((x^3-y^3)*(x^2+y^2-x*y));simplify(%);`

$$\frac{2}{x^3-y^3}+\frac{2x+2y}{(y^2+x^2)^2-y^2x^2}-\frac{2xy}{(x^3-y^3)(x^2+y^2-yx)}$$

$$\frac{4x}{(x^2+y^2-yx)(y^2+yx+x^2)}$$

11] `1/(x^3+x^2-4*x-4)+1/(x^2+3*x+2)+2/(x^2-x-2);simplify(%);`

$$\frac{1}{x^3+x^2-4x-4}+\frac{1}{x^2+3x+2}+\frac{2}{x^2-x-2}$$

$$\frac{3}{x^2-4}$$

12] `(2*a^2-4*a+3)/(a^3+1)+a/(a^2-a+1)-5/(2*(a+1));simplify(%);`

$$\frac{2a^2-4a+3}{a^3+1}+\frac{a}{a^2-a+1}-\frac{5}{2a+2}=\frac{1}{2(a+1)}$$

13] `x/(x+y-2)-x/(x+y+2)+(4*y-8)/(x^2+2*x*y+y^2-4);simplify(%);`

$$\frac{x}{x+y-2}-\frac{x}{x+y+2}+\frac{4y-8}{x^2+2yx+y^2-4}=\frac{4}{x+y+2}$$

14] `a/(a-2)-(2*a)/(a+1)+a/(a-1)+(5*a^2-14)/(a^3-2*a^2-a+2);simplify(%);`

$$\frac{a}{a-2}-\frac{2a}{a+1}+\frac{a}{a-1}+\frac{5a^2-14}{a^3-2a^2-a+2}=\frac{10a^2-7a-14}{a^3-2a^2-a+2}$$

15] `x/(2*x^3+3*x^2+3*x+1)+(x-1)/(2*x+1)-(2-x)/(x^2-4)+(x)/(x^2+x+1)-(1)/(x+2);simplify(%);`

$$\frac{x}{2x^3+3x^2+3x+1}+\frac{x-1}{2x+1}-\frac{2-x}{x^2-4}+\frac{x}{x^2+x+1}-\frac{1}{x+2}=\frac{x^3+2x^2+2x-1}{(2x+1)(x^2+x+1)}$$

16] `1/((x+y)^2-x*y)-(x^2+(x+y)*(y+1))/(x^3-y^3);simplify(%);`

$$\frac{1}{(y+x)^2-yx}-\frac{x^2+(y+x)(y+1)}{x^3-y^3}=\frac{y^2+yx+2y+x^2}{-x^3+y^3}$$

Liceo “Carducci” Volterra - Classi 1A, 1B Scientifico - Francesco Daddi - 9 marzo 2009

Esercizi sulle frazioni algebriche (foglio 2)

Svolgi le seguenti espressioni:

Esercizio 1. $\dfrac{x-2}{16-x^2}+\dfrac{x-1}{x+4}$

Esercizio 2. $\dfrac{x}{x^2-1}+\dfrac{1}{x-1}+\dfrac{2-x}{1-x}+\dfrac{x+1}{1-x^4}$

Esercizio 3. $\dfrac{x-1}{3-x}+\dfrac{x^2+1}{x+3}-\dfrac{x-2}{9-x^2}$

Esercizio 4. $\dfrac{1}{x+2}-\dfrac{x-1}{3-x}-\dfrac{5}{x^2-5\,x+6}-\dfrac{x}{4-x^2}$

Esercizio 5. $\dfrac{6\,x}{x^2-4}+\dfrac{3}{2-x}-\dfrac{1}{x+2}$

Esercizio 6. $\dfrac{x}{y-x}+\dfrac{x}{x+y}-\dfrac{2\,xy}{x^2-y^2}$

Esercizio 7. $-\dfrac{1}{4\,x-x^2-4}-\dfrac{4}{x^2-4}-\dfrac{x}{2-x}-\dfrac{x+3}{x+2}$

Esercizio 8. $\dfrac{x+2}{(x-1)^2}-\dfrac{x+2}{1-x}$

Esercizio 9. $-\dfrac{x}{(x-2)(x-1)}-\dfrac{3}{(x-2)^2}+\dfrac{x-1}{(1-x)(2-x)}+\dfrac{3\,x-3}{(1-x)^2}$

Esercizio 10. $\dfrac{a+1}{a^2-3\,a+2}-\dfrac{a}{(4-a^2)(1-a)}+a^2-a$

Esercizio 11. $\dfrac{x-2}{x^3-y^3-2\,y(x^2-y^2)+(x-y)(x^2+y^2)}+\dfrac{x}{y-x}$

Esercizio 12. $\dfrac{x-1}{x^3+x^2+x+1}+\dfrac{2\,x}{x^3-x^2+x-1}-\dfrac{x-3}{1-x^2}$

Esercizio 13. $\dfrac{1}{2\,x-1-x^2}-\dfrac{x}{1-x}$

Esercizio 14. $\dfrac{(x-1)^2}{x^3-3\,x^2+3\,x-1}-\dfrac{x-1}{(1-x)^3}$

Esercizio 15. $\dfrac{1}{x-2\,a^2}-\dfrac{x-2}{x^3-2\,a(a+3)x^2+(12\,a^3+9\,a^2)x-18\,a^4}+\dfrac{1-x}{3\,a-x}$

Esercizio 16. $\dfrac{x^2}{x^4+x^2+1}-\dfrac{1}{x^2+x+1}$

Soluzioni esercizi 9 marzo 2009 (foglio 2)

1] `(x-2)/(16-x^2)+(x-1)/(x+4);simplify(%);`

$$\frac{x-2}{16-x^2}+\frac{x-1}{x+4}$$

$$\frac{x^2-6x+6}{-16+x^2}$$

2] `(x)/(x^2-1)+(1)/(x-1)+(2-x)/(1-x)+(x+1)/(1-x^4);simplify(%);`

$$\frac{x}{x^2-1}+\frac{1}{x-1}+\frac{2-x}{1-x}+\frac{x+1}{1-x^4}$$

$$\frac{x^3+2x^2+2x+2}{x^3+x^2+x+1}$$

3] `(x-1)/(3-x)+(x^2+1)/(x+3)-(x-2)/(9-x^2);simplify(%);`

$$\frac{x-1}{3-x}+\frac{x^2+1}{x+3}-\frac{x-2}{9-x^2}$$

$$\frac{x^3-4x^2-2}{-9+x^2}$$

4] `(1)/(x+2)-(x-1)/(3-x)-(5)/(x^2-5*x+6)-(x)/(4-x^2);simplify(%);`

$$\frac{1}{x+2}-\frac{x-1}{3-x}-\frac{5}{x^2-5x+6}-\frac{x}{4-x^2}$$

$$\frac{(x^2+x-17)x}{(-4+x^2)(x-3)}$$

5] `6*x/(x^2-4)+3/(2-x)-1/(x+2);simplify(%);`

$$\frac{6x}{-4+x^2}+\frac{3}{2-x}-\frac{1}{x+2}$$

$$\frac{2}{x+2}$$

6] `x/(y-x)+x/(x+y)-(2*x*y)/(x^2-y^2);simplify(%);`

$$\frac{x}{y-x}+\frac{x}{x+y}-\frac{2xy}{x^2-y^2}$$

$$\frac{4xy}{-x^2+y^2}$$

7] `-1/(4*x-x^2-4)-4/(x^2-4)-x/(2-x)-(x+3)/(x+2);simplify(%);`

$$-\frac{1}{4x-x^2-4}-\frac{4}{-4+x^2}-\frac{x}{2-x}-\frac{x+3}{x+2}=\frac{x-1}{(x-2)^2}$$

8] `(x+2)/((x-1)^2)-(x+2)/(1-x);simplify(%);`

$$\frac{x+2}{(x-1)^2}-\frac{x+2}{1-x}$$

$$\frac{x\,(x+2)}{(x-1)^2}$$

9] `-(x)/((x-2)*(x-1))-(3)/((x-2)^2)+(x-1)/((1-x)*(2-x))+(3*x-3)/((1-x)^2);simplify(%);`

$$-\frac{x}{(x-2)\,(x-1)}-\frac{3}{(x-2)^2}+\frac{x-1}{(1-x)\,(2-x)}+\frac{3\,x-3}{(1-x)^2}$$

$$\frac{3\,x^2-16\,x+17}{(x-1)\,(x-2)^2}$$

10] `(a+1)/(a^2-3*a+2)-a/((4-a^2)*(1-a))+a^2-a;simplify(%);`

$$\frac{a+1}{a^2-3\,a+2}-\frac{a}{(4-a^2)\,(1-a)}+a^2-a$$

$$\frac{a^5-2\,a^4-3\,a^3+9\,a^2-2\,a+2}{(a^2-4)\,(a-1)}$$

11] `(x-2)/(x^3-y^3-2*y*(x^2-y^2)+(x-y)*(x^2+y^2))+(x)/(y-x);simplify(%);`

$$\frac{x-2}{x^3-y^3-2\,y\,(x^2-y^2)+(x-y)\,(y^2+x^2)}+\frac{x}{-x+y}=\frac{x^2\,y+x-2-2\,x^3}{(-x+y)\,(y-2\,x)\,x}$$

12] `(x-1)/(x^3+x^2+x+1)+(2*x)/(x^3-x^2+x-1)-(x-3)/(1-x^2);simplify(%);`

$$\frac{x-1}{x^3+x^2+x+1}+\frac{2\,x}{x^3-x^2+x-1}-\frac{x-3}{-x^2+1}=\frac{x^2+x+2}{(x+1)\,(x^2+1)}$$

13] `1/(2*x-1-x^2)-x/(1-x);simplify(%);`

$$\frac{1}{-x^2+2\,x-1}-\frac{x}{-x+1}=\frac{x^2-x-1}{(x-1)^2}$$

14] `((x-1)^2)/(x^3-3*x^2+3*x-1)-(x-1)/((1-x)^3);simplify(%);`

$$\frac{(x-1)^2}{x^3-3\,x^2+3\,x-1}-\frac{x-1}{(-x+1)^3}=\frac{x}{(x-1)^2}$$

15] `1/(x-2*a^2)-(x-2)/(x^3-2*a*(a+3)*x^2+(12*a^3+9*a^2)*x-18*a^4)+(1-x)/(3*a-x);simplify(%);`

$$\frac{1}{x-2\,a^2}-\frac{x-2}{x^3-2\,a\,(a+3)\,x^2+(12\,a^3+9\,a^2)\,x-18\,a^4}+\frac{1-x}{3\,a-x}$$

$$\frac{x^3-3\,a\,x^2-2\,a^2\,x^2+2\,x\,a^2-3\,x\,a-x+6\,x\,a^3+2-6\,a^3+9\,a^2}{(-3\,a+x)\,(x^2-3\,x\,a-2\,x\,a^2+6\,a^3)}$$

16] `x^2/(x^4+x^2+1)-1/(x^2+x+1);simplify(%);`

$$\frac{x^2}{x^4+x^2+1}-\frac{1}{x^2+x+1}=\frac{-1+x}{(x^2+x+1)\,(x^2-x+1)}$$

Esercizi sulle equazioni fratte - Francesco Daddi
18 aprile 2011

Es. 1 $\frac{1}{x^2-3x+2}+\frac{2}{x-1}=0$ sol: $x=\frac{3}{2}$

Es. 2 $\frac{4x-3}{x^2-4}-\frac{3x}{x-2}=\frac{4}{2-x}-\frac{4x}{2+x}$ sol: $x_1=1$; $x_2=5$

Es. 3 $\frac{3x+2}{2x^2-2x-12}-\frac{3-x}{4x-12}=-\frac{3}{x+2}$ sol: $x_1=-19$; $x_2=2$

Es. 4 $\frac{2x-1}{3x^2-75}-\frac{3-x}{x+5}+\frac{x-3}{10-2x}=\frac{7}{25-x^2}$ sol: $x=\frac{35}{3}$

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Es. 10 $\frac{5x}{3x^2-18x+15}-\frac{2}{3x-3}=\frac{5}{18x-90}$ sol: $x=-5$

Es. 11 $\frac{2x}{x^2+2x-8}-\frac{2x+7}{x^2-3x-4}=0$ sol: $x_1=-2$; $x_2=\frac{28}{17}$

Es. 12 $\frac{1-x}{x^2-4x+3}-\frac{4}{9-x^2}+\frac{x-3}{x^2+4x+3}=-\frac{5}{3-x}$ sol: $x_1=-5$; $x_2=-\frac{1}{5}$

Es. 13 $\frac{4x-7}{x+2}+\frac{1-6x^2}{x^2-5x+6}=\frac{x}{2x^2-2x-12}-2$ sol: Equazione impossibile

Es. 14 $\frac{1}{x-2}+\frac{2}{(x-2)^2}=\frac{3}{(x-2)^3}$ sol: $x_1=-1$; $x_2=3$

Es. 15 $\frac{1}{x+3}-\frac{5(x+2)}{(x+3)^2}=\frac{5x-1}{(x+3)^3}$ sol: $x_1=-5$; $x_2=-1$

Es. 16 $\frac{3}{(3x-6)^2}-\frac{x^2-4}{(3x-6)^4}$ sol: $x=\frac{28}{13}$

Es. 17 $\frac{2x}{x^2-2x+1}=\frac{-7}{3x^2-21x+18}+\frac{2x}{x^2-3x+2}$ sol: $x_1=-14$; $x_2=-1$

Es. 18 $\frac{5x-3}{x^2-5x}+\frac{2}{x}=\frac{3x}{x^2+3x}-\frac{2}{x+3}-\frac{4}{5-x}$

sol: $x_1=\frac{-1-\sqrt{313}}{4}$; $x_2=\frac{-1+\sqrt{313}}{4}$

Es. 19 $\frac{x-9}{4x-x^2}-\frac{3x+2}{2-x}=\frac{x-5}{x+2}+\frac{2x^4+6x^3}{x(x-4)(x^2-4)}$ sol: Equaz. impossibile

Es. 20 $\frac{3-3x}{x^2-1}+\frac{8x}{2-2x}=0$ sol. $x_1=\frac{-7-\sqrt{97}}{8}$; $x_2=\frac{-7+\sqrt{97}}{8}$

Francesco Daddi - 1 ottobre 2009

Esercizio svolto: equazione fratta

Esercizio 1. *Risolvere l'equazione*

$$\frac{x-4}{x+1}+\frac{1}{x-3}=2\,x+\frac{28}{3}\ .$$

Soluzione. Il dominio dell'equazione è $x\neq -1$, $x\neq 3$.
Portiamo tutti i termini a sinistra:

$$\frac{x-4}{x+1}+\frac{1}{x-3}-2\,x-\frac{28}{3}=0\ ;$$

scriviamo tutto con un unico denominatore:

$$\frac{3(x-3)(x-4)+3(x+1)+3(x+1)(x-3)(-2\,x)+(x+1)(x-3)(-28)}{3(x+1)(x-3)}=0$$

svolgendo e semplificando otteniamo:

$$\frac{-6\,x^3-13\,x^2+56\,x+123}{3(x+1)(x-3)}=0\ .$$

Studiamo ora le radici del polinomio $-6\,x^3-13\,x^2+56\,x+123$; si osserva che una radice del polinomio è $x_1=-3$ (accettabile), per cui è possibile effettuare la divisione polinomiale:

$$-6\,x^3-13\,x^2+56\,x+123=(x+3)(-6\,x^2+5\,x+41)\ ;$$

a questo punto determiniamo le soluzioni dell'equazione $-6\,x^2+5\,x+41=0$:

$$\begin{cases} a=-6 \\ b=5 \\ c=41 \end{cases} \quad\Rightarrow\quad \Delta=b^2-4\cdot a\cdot c=(5)^2-4\cdot(-6)\cdot 41=1009 \quad\Rightarrow$$

$$x_{2,3}=\frac{-b\pm\sqrt{\Delta}}{2\cdot a}=\frac{-5\pm\sqrt{1009}}{2\cdot(-6)}=\frac{-5\pm\sqrt{1009}}{-12}\ \begin{array}{l}\nearrow\ x_2=\dfrac{-5+\sqrt{1009}}{-12}=\dfrac{5-\sqrt{1009}}{12}\\ \searrow\ x_3=\dfrac{-5-\sqrt{1009}}{-12}=\dfrac{5+\sqrt{1009}}{12}\end{array}\ ;$$

si osservi che anche questi valori sono accettabili.
In definitiva, le soluzioni dell'equazione iniziale sono

$$x_1=-3\ \ ;\ \ x_2=\frac{5-\sqrt{1009}}{12}\approx -2.2304\ \ ;\ \ x_3=\frac{5+\sqrt{1009}}{12}\approx 3.0637\ .$$

Esercizio svolto sulle equazioni fratte

Francesco Daddi - 25 maggio 2010

Risolvere la seguente equazione nell'incognita x: $\frac{x+1}{x+3}-\frac{3-2x}{x-2}=3$.

Soluzione. Poniamo $\begin{cases} x+3\neq 0 \\ x-2\neq 0 \end{cases} \rightarrow \begin{cases} x\neq -3 \\ x\neq 2 \end{cases}$; il dominio dell'equazione data, pertanto, è $D=R-\{2\,;-3\}$; svolgendo i calcoli si trova:

$$\frac{(x+1)(x-2)-(3-2x)(x+3)}{(x+3)(x-2)}=\frac{3(x+3)(x-2)}{(x+3)(x-2)}$$

$$x^2-2x+x-2-(3x+9-2x^2-6x)=3(x^2-2x+3x-6)$$

$$x^2-x-2-(-3x+9-2x^2)=3(x^2+x-6)$$

$$x^2-x-2+3x-9+2x^2=3x^2+3x-18$$

$$3x^2+2x-11=3x^2+3x-18$$

$$2x-3x=11-18$$

$$-x=-7$$

$$x=7$$

la soluzione è accettabile.

Esercizi sulle equazioni fratte 1A, 1B Scientifico 9 maggio 2009

Prof. Francesco Daddi

Esercizio 1 $\frac{x+3}{x-2}+x=9$

R. $7, 3$

Esercizio 2 $\frac{x}{x+1}-\frac{1}{x-1}=1$

R. 0

Esercizio 3 $\frac{x^2-1}{x-1}-\frac{1}{x+2}=\frac{x+7}{x+2}-x$

R. -3

Esercizio 4 $\frac{x-1}{x^2-4}=-\frac{5}{x+2}$

R. $\frac{11}{6}$

Esercizio 5 $\frac{x^2-1}{x-1}-1=2x+1$

R. -1

Esercizio 6 $\frac{x-2}{x+3}-x=-6+2x$

R. $\frac{-8}{3}, 2$

Esercizio 7 $4-x^2=\frac{x^2+5x+6}{x+2}-1$

R. 1

Esercizio 8 $\frac{6+x}{x-3}=\frac{x^2}{x-3}$

R. -2

Esercizio 9 $\frac{x^3+2}{x}=2x+1$

R. $1, 2, -1$

Esercizio 10 $-\frac{x^2-4}{x+2}=\frac{x^3-8}{x^2-6x+8}$

R. Nessuna soluzione

Esercizi sulle equazioni intere e fratte (11 maggio 2009)

$\frac{4}{3}-\frac{2x}{3}=x-\frac{5}{2}$ R. $\frac{23}{10}$

$\frac{x-5}{x-4}+\frac{1}{2}=x-5$ R. $\frac{9}{2}, 6$

$(x+3)(2-x)-(-1+x)^2=2$ R. $-1, \frac{3}{2}$

$\frac{1-3x}{(-1+x)^2}-\frac{x}{1-x^2}=\frac{1}{1-x}$ R. $0, -3$

$\frac{4-3x}{x+2}-\frac{5(-1+x)}{8-2x^2}=0$ R. $3, \frac{7}{6}$

$\frac{x-2}{2-x}+x=1$ R. impossibile

$\frac{x}{1-x^3}+\frac{2(-1+x)}{x^2+x+1}=0$ R. $2, \frac{1}{2}$

$\frac{1}{2}x-x^2=x-5$ R. $\frac{-5}{2}, 2$

$\frac{4}{3x}=2x-\frac{2}{3}$ R. $\frac{-2}{3}, 1$

$x^6+6x^4+13x^2+32=0$ R. impossibile

$(x-8)(x^2-25)(x^2-x-12)=0$ R. $8, 5, -5, 4, -3$

$(x^2-4x-5)^2$ R. $5, -1$

$\frac{x}{x-4}=\frac{-1+x^2}{x^2-x}$ R. $\frac{-4}{3}$

Esercizi sulle equazioni fratte - Francesco Daddi
18 aprile 2011

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Esercizi sulle equazioni - 18 luglio 2009

Esercizio 1

$$(x-3)^2+(2-x)^2=x^2+4$$

R. $9, 1$

Esercizio 2

$$(x-2)^3-(1-x)^3-2x^3+3=0$$

R. $\frac{2}{3}, 1$

Esercizio 3

$$\frac{1-x}{x-2}-\frac{4}{4-x^2}=0$$

R. -3

Esercizio 4

$$\frac{(x-2)^2}{(x+3)^2}-\frac{1}{x^2+6x+9}=1$$

R. $\frac{-3}{5}$

Esercizio 5

$$-\frac{2}{3x-3}-\frac{3}{x-1}=\frac{1}{2x-x^2-1}$$

R. $\frac{14}{11}$

Esercizio 6

$$\frac{2-x}{(x+2)^2}+\frac{8}{-4+2x}=0$$

R. $\frac{-2}{3}, -6$

Esercizio 7

$$\frac{1}{x+2}-\frac{x}{x^2-3x-10}=\frac{4}{x-5}$$

R. $\frac{-13}{4}$

Esercizio 8

$$(2x-1)^4+(1+2x)^4=32x^4+50x^2$$

R. $-1, 1$

Esercizio 9

$$\frac{1}{(x-1)^3}=\frac{-1}{8}$$

R. -1

Esercizio 10

$$\frac{(x-3)^2}{(x+2)^2}=25$$

R. $\frac{-13}{4}, \frac{-7}{6}$

Esercizi sulle equazioni - 6 agosto 2009

Esercizio 1

$$\frac{(x+1)^2}{2} - 2x^2 = 0$$

R. $\frac{-1}{3}, 1$

Esercizio 2

$$(1-x)^4 = -x^2 - 1$$

R. impossibile

Esercizio 3

$$(x-2)^2 - x - 3 = (1-x)^2 - x + 3$$

R. $\frac{-3}{2}$

Esercizio 4

$$\frac{1}{x+1} + \frac{2x}{1-x^2} = \frac{2x}{1-x}$$

R. $\frac{1}{2}$

Esercizio 5

$$\frac{-1+x}{x^3-1} - \frac{2}{x^2+x+1} = 0$$

R. impossibile

Esercizio 6

$$\frac{x}{3}+\frac{2\,x}{2\,x+6}=0$$

R. $0, -6$

Esercizio 7

$$\left(-\frac{1}{3}+\frac{x}{3}\right)^2-9=0$$

R. $10, -8$

Esercizio 8

$$x^4-5\,x^2+4=0$$

R. $1, 2, -2, -1$

Esercizio 9

$$799\,x^2+800\,x=-1$$

R. $\frac{-1}{799}, -1$

Esercizio 10

$$\frac{(x+2)^3}{(-1+x)^3}=27$$

R . $\frac{5}{2}$