SOLUZIONI del 23/11/2021

1) $$I_o = \underbrace{\frac{1}{3} M L^2}_{I_{BARRA}} + \underbrace{MR^2 + M\cdot(3R)^2}_{I_{ANELLO}} \Rightarrow$$

$$I_o = \frac{1}{3} M (2R)^2 + MR^2 + M\cdot(9R^2) = \frac{34}{3} MR^2 .$$

$$E_c = \frac{1}{2} I_o \omega^2 \Rightarrow \omega = \sqrt{\frac{2\cdot E_c}{I_o}}$$

$$L = I_o \omega = I_o \cdot \sqrt{\frac{2E_c}{I_o}} = \sqrt{2 I_o E_c} .$$

$$L = \sqrt{2\cdot\left(\frac{34}{3} MR^2\right)\cdot E_c} = \boxed{213 \ kg\cdot \frac{m^2}{s}}$$

2) $$mgh = \frac{1}{2} m v_o^2 \Rightarrow v_o = \sqrt{2gh} .$$

$$m v_o \cdot \frac{L}{2} = \left(\frac{1}{12} ML^2 + m\cdot\left(\frac{L}{2}\right)^2\right)\cdot \omega_f \Rightarrow$$

$$\omega_f = \frac{m\sqrt{2gh}\cdot L/2}{\frac{1}{12} ML^2 + mL^2/4}$$

$$\frac{1}{2} I_o^* \cdot \omega_i^2 + \cancel{mg\cdot 0} = \cancel{\frac{1}{2} I_o^* \cdot 0^2} + mg\cdot\frac{L}{2}$$

$$\Rightarrow \frac{1}{2}\cdot I_o^* \cdot \omega_i^2 = mg\frac{L}{2}$$

$$\frac{1}{2}\cdot\left(\frac{1}{12} ML^2 + m\frac{L^2}{4}\right)\cdot\left(\frac{m\sqrt{2gh}\cdot L/2}{\frac{1}{12} ML^2 + mL^2/4}\right)^2 = mg\frac{L}{2}$$

$$\cancel{\frac{1}{2}}\cdot\frac{m^{\cancel{2}}\cdot(2\cancel{g}h)\cdot L^2/4}{\frac{1}{12} ML^2 + mL^2/4} = \cancel{\frac{1}{2}}\cdot \cancel{mg}L \Rightarrow$$

$$\boxed{h = \left(\frac{1}{2} + \frac{M}{6m}\right)\cdot L} \qquad \boxed{h = 5{,}33 m}$$

3) $$m v_0 \cdot R\cos\theta = \underbrace{\left(\frac{1}{2}MR^2 + mR^2\right)}_{I_0^*}\omega_f$$

$$\omega_f = \frac{m v_0 R\cos\theta}{\frac{1}{2}MR^2 + mR^2} \Rightarrow \boxed{\omega_f = \frac{m v_0 \cos\theta}{R\cdot\left(\frac{M}{2}+m\right)}} \quad \omega_f = 0{,}300\ \frac{rad}{s}$$

- $$\frac{1}{2} I_0^* \omega_i^2 + mg\cdot(-R) = \cancel{\frac{1}{2} I_0^* \cdot 0^2} + mg\cdot(-R\cos\beta)$$

$$\frac{1}{2}\cdot\left(\frac{1}{2}MR^2 + mR^2\right)\cdot\left(\frac{m v_0 \cos\theta}{R\left(\frac{M}{2}+m\right)}\right)^2 = mgR(1-\cos\beta)$$

$$\frac{\cancel{R^2}}{2}\cdot\cancel{\left(\frac{1}{2}M + m\right)}\cdot\frac{m^{\cancel{2}} v_0^2 \cos^2\theta}{\cancel{R^2}\cdot\left(\frac{M}{2}+m\right)^{\cancel{2}}} = \cancel{m}gR(1-\cos\beta)$$

$$\frac{m v_0^2 \cos^2\theta}{M+2m} = gR(1-\cos\beta)$$

$$\boxed{\cos\beta = 1 - \frac{m v_0^2 \cos^2\theta}{gR(M+2m)}} \Rightarrow \boxed{\beta = 7{,}23^\circ}$$

- $$I_0^* \alpha = -mgR\sin\beta \Rightarrow I_0^*\alpha = -mgR\cdot\beta$$

$$\alpha = -\frac{mgR}{I_0^*}\cdot\beta \quad ; \quad \omega^2 = \frac{mgR}{I_0^*}$$

$$T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\cdot\sqrt{\frac{I_0^*}{mgR}} = 2\pi\cdot\sqrt{\frac{\frac{1}{2}MR^2 + mR^2}{mgR}}$$

$$\boxed{T = 2\pi\sqrt{\left(\frac{M}{2m}+1\right)\frac{R}{g}}} \Rightarrow T = 2{,}64\ s.$$