SOLUZIONI – 5/11/2021

DOMINIO $\boxed{0 \leq x \leq \frac{\pi}{2}}$

1) $\overline{AP} = 2r\cos x$

$\overline{PC} = \overline{HD} = \overline{AD} - \overline{AH}$

dove H è la proiez. ort. di P su AB.

$\overline{AD} = 3r$; $\overline{AH} = \overline{AP} \cdot \cos x = 2r\cos^2 x$

QUINDI $\overline{PC} = 3r - 2r\cos^2 x$.

$$f(x) = \overline{AP} + \overline{PC} = 2r\cos x + 3r - 2r\cos^2 x$$

$$\Rightarrow f(x) = 2r \cdot \left(-\cos^2 x + \cos x + \frac{3}{2}\right) =$$

$$= 2r \cdot \left[-\left(\cos x - \frac{1}{2}\right)^2 + \frac{1}{4} + \frac{3}{2}\right] =$$

$$= 2r \cdot \left[\frac{7}{4} - \left(\cos x - \frac{1}{2}\right)^2\right]$$

IL MAX SI OTTIENE QUANDO $\cos x - \frac{1}{2} = 0$

$$\Rightarrow \boxed{x = \frac{\pi}{3}} \qquad \boxed{f_{MAX} = \frac{7}{2}r}$$

2) $f(x) = \overline{PC} - \overline{PB}$; $\overline{PB} = 2r\sin x$

$\boxed{0 \leq x \leq \frac{\pi}{2}}$

$$f(x) = 3r - 2r\cos^2 x - 2r\sin x =$$

$$= 3r - 2r(1 - \sin^2 x) - 2r\sin x =$$

$$= r + 2r\sin^2 x - 2r\sin x =$$

$$= r + 2r \cdot \left[\left(\sin x - \frac{1}{2}\right)^2 - \frac{1}{4}\right] = \frac{r}{2} + 2r\left(\sin x - \frac{1}{2}\right)^2$$

IL MINIMO SI OTTIENE PER $\sin x - \frac{1}{2} = 0 \longrightarrow$

$$\boxed{x = \frac{\pi}{6}} \qquad \boxed{f_{MIN} = \frac{r}{2}}$$

3) $f(x) = \overline{PH} + \overline{HB}$ $\boxed{0 \leq x \leq \frac{\pi}{2}}$

$\overline{PH} = \overline{AP} \cdot \sin x = (2r\cos x) \cdot \sin x$

$\overline{HB} = \overline{BP} \cdot \sin x = (2r\sin x) \cdot \sin x$

$$f(x) = \underbrace{2r\cos x \sin x}_{\overline{PH}} + \underbrace{2r\sin^2 x}_{\overline{HB}}$$

$$\Rightarrow f(x) = 2r(\sin x \cos x + \sin^2 x)$$

$$\begin{cases} \sin x \cos x = \frac{1}{2}\sin(2x) \ ; \\ \sin^2 x = \frac{1-\cos(2x)}{2} \end{cases}$$

$$f(x) = 2r \cdot \left(\frac{1}{2}\sin(2x) + \frac{1}{2} - \frac{1}{2}\cos(2x)\right) =$$

$$= r + r \cdot (\sin(2x) - \cos(2x)) =$$

$$= r + \sqrt{2}\, r \cdot \left(\underbrace{\frac{1}{\sqrt{2}}}_{\cos\varphi}\sin(2x) - \underbrace{\frac{1}{\sqrt{2}}}_{\sin\varphi}\cos(2x)\right)$$

$\varphi = -\frac{\pi}{4}$

QUINDI $f(x) = r + \sqrt{2}\, r \cdot \sin\left(2x - \frac{\pi}{4}\right)$

IL MAX SI OTTIENE QUANDO $2x - \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi$

$x = \frac{3}{8}\pi + k\pi$ ACCETT. SOLO CON $k = 0$

QUINDI $\boxed{x = \frac{3}{8}\pi}$ $\boxed{f_{MAX} = r \cdot (1 + \sqrt{2})}$.

4)

$$\overline{AP} = 2r \cdot \cos x$$

$$\overline{AQ} = 2\sqrt{3}\, r \cdot \cos x$$

$$\overline{QC} = 2\sqrt{3}\, r \cdot \sin x$$

DOMINIO

$$\boxed{0 \leq x \leq \frac{\pi}{2}}$$

$$f(x) = \overline{PQ} + \overline{QC}$$

$$f(x) = (\overline{AQ} - \overline{AP}) + \overline{QC}$$

QUINDI:

$$f(x) = (2\sqrt{3}\, r \cos x - 2r\cos x) + 2\sqrt{3}\, r \sin x$$

$$f(x) = 2r \cdot \left[(\sqrt{3}-1)\cos x + \sqrt{3}\sin x\right]$$

$$\sqrt{(\sqrt{3}-1)^2 + (\sqrt{3})^2} = \sqrt{7-2\sqrt{3}} = A.$$

QUINDI, CON LA TECNICA DELL'ANGOLO AGGIUNTO, SI HA:

$$f(x) = 2r \cdot A \cdot \left(\frac{\sqrt{3}-1}{A}\cos x + \frac{\sqrt{3}}{A}\sin x\right)$$

$$f(x) = 2r \cdot \sqrt{7-2\sqrt{3}} \cdot \sin\left(x + \underbrace{\arctan\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)}_{\varphi}\right)$$

IL MAX SI OTTIENE PER

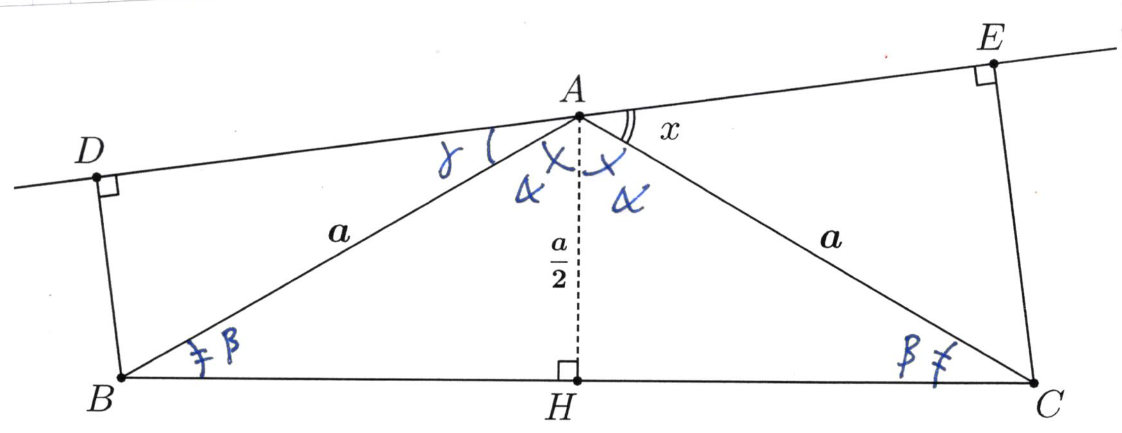
$$x + \varphi = \frac{\pi}{2} + 2k\pi \quad \Rightarrow \quad x = \frac{\pi}{2} - \varphi + 2k\pi$$

L'UNICA SOLUZIONE ACCETTABILE SI TROVA CON $K=0$:

$$x = \frac{\pi}{2} - \arctan\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \quad \text{OVVERO}$$

$$x = \arctan\left(\frac{\sqrt{3}}{\sqrt{3}-1}\right) = \boxed{\arctan\left(\frac{3+\sqrt{3}}{2}\right) \approx 67{,}09^\circ}$$

5)



$\alpha = 60° \left(= \frac{\pi}{3} \text{ rad}\right)$; $\beta = 30° \left(= \frac{\pi}{6} \text{ rad}\right)$.

$\boxed{0 \leq x \leq \frac{\pi}{3}}$ $\quad \overline{BH} = \frac{\sqrt{3}}{2} a \Rightarrow \overline{BC} = \sqrt{3}\, a$.

$$\begin{cases} \overline{EC} = a \cdot \sin x \\ \overline{AE} = a \cdot \cos x \end{cases} \qquad \begin{cases} \overline{AD} = a \cdot \cos\gamma \\ \overline{BD} = a \cdot \sin\gamma \end{cases}$$

$$\gamma + \alpha + \alpha + x = \pi \Rightarrow \gamma = \pi - x - \frac{2}{3}\pi$$

$$\Rightarrow \gamma = \frac{\pi}{3} - x$$

$$f(x) = \overline{BC} + \overline{EC} + \underbrace{\overline{ED}}_{\overline{AD} + \overline{AE}} + \overline{DB}$$

QUINDI:

$$f(x) = \sqrt{3}\, a + a \sin x + a \cos\gamma + a \cos x + a \sin\gamma$$

essendo: $\begin{cases} \cos\gamma = \cos\left(\frac{\pi}{3} - x\right) = \ldots = \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x \\ \sin\gamma = \sin\left(\frac{\pi}{3} - x\right) = \ldots = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x \end{cases}$

si ricava:

$$f(x) = \sqrt{3}\, a + \frac{1+\sqrt{3}}{2} a \sin x + \frac{3+\sqrt{3}}{2} a \cos x$$

$$f(x) = \sqrt{3}\,a + \frac{1+\sqrt{3}}{2}\,a \sin x + \frac{(1+\sqrt{3})\sqrt{3}}{2}\,a \cos x$$

$$\Rightarrow f(x) = \sqrt{3}\,a + (1+\sqrt{3})\,a \cdot \left[\underbrace{\frac{1}{2}}_{\cos\varphi} \sin x + \underbrace{\frac{\sqrt{3}}{2}}_{\sin\varphi} \cos x \right]$$

$$\begin{cases} \cos\varphi = \frac{1}{2} \\ \sin\varphi = \sqrt{3}/2 \end{cases} \Rightarrow \varphi = \frac{\pi}{3}$$

QUINDI ABBIAMO:

$$f(x) = \sqrt{3}\,a + (1+\sqrt{3})\,a \cdot \sin\left(x + \frac{\pi}{3}\right)$$

IL MAX SI OTTIENE QUANDO $x + \frac{\pi}{3} = \frac{\pi}{2} + 2k\pi$

$$\Rightarrow x = \frac{\pi}{6} + 2k\pi$$

È ACCETTABILE SOLO CON $k=0$:

$$\boxed{x = \frac{\pi}{6}} \qquad \boxed{f_{MAX} = (1+2\sqrt{3})\,a}$$