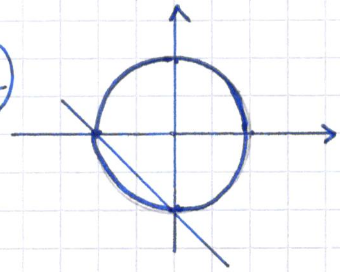
SOLUZ. 8/11/2021

1)



$$\sin x + \cos x = -1$$

$$Y + X = -1 \Rightarrow Y = -X - 1$$

$$\boxed{x = \pi + 2k\pi \quad \vee \quad x = -\frac{\pi}{2} + 2k\pi}$$

2) $$2\sin^2 x - (\sqrt{3}+1)\sin x \cos x + (\sqrt{3}+1)\cos^2 x = \underset{\sin^2 x + \cos^2 x}{\underbrace{1}}$$

$$\sin^2 x - (\sqrt{3}+1)\sin x \cos x + \sqrt{3}\cos^2 x = 0$$

OMOGENEA di 2° GRADO → DIVIDO PER $\cos^2 x$:

$$\tan^2 x - (\sqrt{3}+1)\tan x + \sqrt{3} = 0$$

$$\Delta = 3 + 1 + 2\sqrt{3} - 4\sqrt{3} = 3 + 1 - 2\sqrt{3} = (\sqrt{3}-1)^2$$

$$\tan x = \frac{\sqrt{3}+1 \pm (\sqrt{3}-1)}{2 \cdot 1} = \begin{cases} \sqrt{3} \\ 1 \end{cases}$$

$$\tan x = \sqrt{3} \rightarrow \boxed{x = \frac{\pi}{3} + k\pi}$$

$$\tan x = 1 \rightarrow \boxed{x = \frac{\pi}{4} + k\pi}.$$

3) $$\sin x \cdot (2\sin^2 x - 5\cos x + 1) = 0$$

$$\sin x = 0 \rightarrow \boxed{x = k\pi}$$

$$2(1 - \cos^2 x) - 5\cos x + 1 = 0$$

$$2\cos^2 x + 5\cos x - 3 = 0$$

$$(2\cos x - 1)(\cos x + 3) = 0$$

$$2\cos x - 1 = 0 \rightarrow \boxed{x = \pm\frac{\pi}{3} + 2k\pi}$$

$\cos x + 3 = 0 \rightarrow \nexists$ SOLUZ. REALI

4) $\sqrt{3}\sin^4 x - 4\sin^5 x\cos x = 0$

$$\sin^4 x \cdot (\sqrt{3} - 4\underbrace{\sin x\cos x}_{\frac{\sin(2x)}{2}}) = 0$$

$\sin^4 x = 0$

$\boxed{x = k\pi}$

$\sqrt{3} - 2\sin(2x) = 0$

$\sin(2x) = \frac{\sqrt{3}}{2}$

$2x = \frac{\pi}{3} + 2k\pi \rightarrow \boxed{x = \frac{\pi}{6} + k\pi}$

$2x = \frac{2}{3}\pi + 2k\pi \rightarrow \boxed{x = \frac{\pi}{3} + k\pi}$

5) $y = 3 \rightarrow$ RETTA DEI FLESSI ; $A = 5$; $T = \frac{\pi}{2}$

$y = -\sin x$ (F. di PARTENZA).

$$\varphi: \begin{cases} x' = \frac{1}{4}x + x_F \\ y' = 5y + y_F \end{cases} \quad \text{CON } F\left(\frac{\pi}{3}; 3\right)$$

$\frac{\pi/2}{2\pi}$

$$\varphi: \begin{cases} x' = \frac{x}{4} + \frac{\pi}{3} \\ y' = 5y + 3 \end{cases} \Rightarrow \varphi^{-1}: \begin{cases} x = \frac{x' - \pi/3}{1/4} = 4\left(x' - \frac{\pi}{3}\right) \\ y = \frac{y' - 3}{5} \end{cases}$$

$$\Rightarrow \frac{y' - 3}{5} = -\sin\left(4x' - \frac{4}{3}\pi\right) \Rightarrow$$

$$y' = 3 - 5\sin\left(4x' - \frac{4}{3}\pi\right)$$

TOGLIENDO GLI APICI : $\boxed{y = 3 - 5\sin\left(4x - \frac{4}{3}\pi\right)}$

MAX : $x = \frac{5}{24}\pi + k\frac{\pi}{2}$; MIN: $x = \frac{11}{24}\pi + k\frac{\pi}{2}$; FLESSI: $x = \frac{\pi}{3} + k\pi/4$

6) $a = \frac{-6+14}{2} = 4$; $A = \frac{14-(-6)}{2} = 10.$

$A = \sqrt{b^2 + 6^2} \Rightarrow 10 = \sqrt{b^2 + 36} \Rightarrow$

$b^2 = 64 \Rightarrow b = \pm 8.$

$f_1(x) = 4 + 8\cos(\dots) + 6\sin(\dots)$

$f_2(x) = 4 - 8\cos(\dots) + 6\sin(\dots)$

7) $f(x) = \cos\left(2x + \frac{\pi}{6}\right) + \sin(2x - \pi)$

SVOLGENDO TROVIAMO:

$f(x) = \frac{\sqrt{3}}{2}\cos(2x) - \frac{3}{2}\sin(2x) \Rightarrow$

$f(x) = \sqrt{3}\cdot\left[-\frac{\sqrt{3}}{2}\sin(2x) + \frac{1}{2}\cos(2x)\right] \Rightarrow$

$\boxed{f(x) = \sqrt{3}\cdot\sin\left(2x + \frac{5}{6}\pi\right)}.$

MAX $\rightarrow 2x + \frac{5}{6}\pi = \frac{\pi}{2} + 2k\pi \Rightarrow x = -\frac{\pi}{6} + k\pi$

MIN $\rightarrow 2x + \frac{5}{6}\pi = -\frac{\pi}{2} + 2k\pi \Rightarrow x = -\frac{2}{3}\pi + k\pi$

FLESSI $\rightarrow 2x + \frac{5}{6}\pi = k\pi \Rightarrow x = -\frac{5}{12}\pi + \frac{k\pi}{2}.$

8) $f(x) = \sin^2 x + a\cos^2 x + b$

$f(x) = (1 - \cos^2 x) + a\cos^2 x + b$

$f(x) = b + 1 + (a-1)\cos^2 x$

1° CASO: $a > 1 \Rightarrow \begin{cases} b+1 = 4 \\ a-1 = 9-4 \end{cases} \Rightarrow \begin{cases} b = 3 \\ a = 6 \end{cases}$

2° CASO: $a < 1 \Rightarrow \begin{cases} b+1 = 9 \\ a-1 = 4-9 \end{cases} \Rightarrow \begin{cases} b = 8 \\ a = -4 \end{cases}$

$$\boxed{f(x) = \sin^2 x + 6\cos^2 x + 3}$$

OPPURE

$$\boxed{f(x) = \sin^2 x - 4\cos^2 x + 8}$$

2° METODO:

$y = 1 + b + (a-1)\cos^2 x \Rightarrow$

$y = \underbrace{1 + b + \frac{a-1}{2}}_{=(4+9)/2} + \frac{a-1}{2}\cos(2x)$

$\downarrow$

$\frac{|a-1|}{2} = \frac{9-4}{2}$

9) $f(x) = 2\cos^2 x - 3\cos x = 2\left(\cos^2 x - \frac{3}{2}\cos x\right) \Rightarrow$

$f(x) = 2\cdot\left[\left(\cos x - \frac{3}{4}\right)^2 - \frac{9}{16}\right] = 2\cdot\left(\cos x - \frac{3}{4}\right)^2 - \frac{9}{8}.$

MINIMO ASSOLUTO: $\cos x - \frac{3}{4} = 0 \Rightarrow \cos x = \frac{3}{4}$

$\Rightarrow \quad x = \pm \arccos\left(\frac{3}{4}\right) + 2k\pi$

MASSIMO ASSOLUTO: $\left|\cos x - \frac{3}{4}\right|$ MAX

SI OTTIENE IN CORRISPONDENZA di $\cos x = -1$:

$x = \pi + 2k\pi.$

MASSIMI RELATIVI NON ASSOLUTI: $\cos x = 1$:

$x = 2k\pi.$

10) $y = 2$: retta dei FLESSI.

CON LE F. di DUPLICAZIONE:
$y = a + b\cdot\frac{1+\cos(2x)}{2} + c\cdot\frac{\sin(2x)}{2}$

$A = 2$; PASSAGGIO PER $P\left(\frac{\pi}{4}; 3\right)$

$$\begin{cases} a + \frac{b}{2} = 2 \\ \left(\frac{b}{2}\right)^2 + \left(\frac{c}{2}\right)^2 = 2^2 \\ a + b\cdot\underbrace{\cos^2\left(\frac{\pi}{4}\right)}_{=1/2} + c\cdot\underbrace{\sin\left(\frac{\pi}{4}\right)}_{=1/\sqrt{2}}\cdot\underbrace{\cos\left(\frac{\pi}{4}\right)}_{=1/\sqrt{2}} = 3 \end{cases} \Longrightarrow$$

$$\begin{cases} a + \frac{b}{2} = 2 \\ b^2 + c^2 = 16 \\ \underbrace{a + \frac{b}{2}}_{=2} + \frac{c}{2} = 3 \end{cases} \Rightarrow \begin{cases} a = 2 + \sqrt{3} \\ b = -2\sqrt{3} \\ c = 2 \end{cases} \quad (b < 0) \text{ N.B.}$$

$$y = 2 + \sqrt{3} - 2\sqrt{3}\cos^2 x + 2\underbrace{\sin x \cos x}$$

$$\cos^2 x \to \frac{1+\cos(2x)}{2} \qquad \sin x \cos x \to \frac{\sin(2x)}{2}$$

$$y = 2 + \sqrt{3} - \sqrt{3}\left(1 + \cos(2x)\right) + \sin(2x)$$

$$\Rightarrow y = 2 + \cancel{\sqrt{3}} - \cancel{\sqrt{3}} - \sqrt{3}\cos(2x) + \sin(2x)$$

$$\Rightarrow y = 2 + 2\cdot\left[\frac{1}{2}\sin(2x) - \underbrace{\frac{\sqrt{3}}{2}}\cos(2x)\right]$$

$$\frac{1}{2} \to \cos\varphi \qquad \frac{\sqrt{3}}{2} \to \sin\varphi$$

$$\varphi = -\frac{\pi}{3}$$

$$\boxed{y = 2 + 2\sin\left(2x - \frac{\pi}{3}\right)}$$

MAX $\longrightarrow$ $2x - \frac{\pi}{3} = \frac{\pi}{2} + 2k\pi \longrightarrow x = \frac{5}{12}\pi + k\pi$

MIN $\longrightarrow$ $2x - \frac{\pi}{3} = -\frac{\pi}{2} + 2k\pi \longrightarrow x = -\frac{\pi}{12} + k\pi$

FLESSI $\longrightarrow$ $2x - \frac{\pi}{3} = k\pi \longrightarrow x = \frac{\pi}{6} + k\frac{\pi}{2}$.

11) $\overline{AP} = 2r\cos x$;

$$\boxed{0 \leq x \leq \frac{\pi}{2}}$$

$$\overline{PH} = \overline{KB} = \underbrace{\overline{AB}}_{2r} - \underbrace{\overline{AK}}_{\overline{AP}\cdot\cos x}$$

$$\overline{PH} = 2r - 2r\cos^2 x = 2r(1-\cos^2 x)$$

$$f(x) = 4r^2\cdot\left[\cos^2 x + 2\cdot(1-\cos^2 x)^2\right] \Rightarrow$$

$$f(x) = 4r^2\cdot\left[2\cos^4 x - 3\cos^2 x + 2\right]$$

QUINDI, CON IL COMPLETAMENTO DEL QUADRATO, SI RICAVA:

$$f(x) = 4r^2 \cdot \left[2\left(\cos^4 x - \frac{3}{2}\cos^2 x + 1\right)\right] \Rightarrow$$

$$f(x) = 8r^2 \cdot \left[\left(\cos^2 x - \frac{3}{4}\right)^2 - \frac{9}{16} + 1\right] \Rightarrow$$

$$f(x) = 8r^2 \cdot \left[\left(\cos^2 x - \frac{3}{4}\right)^2 + \frac{7}{16}\right] \Rightarrow$$

$$\Rightarrow \boxed{f(x) = \frac{7}{2}r^2 + 8r^2\left(\cos^2 x - \frac{3}{4}\right)^2}.$$

MINIMO: $\cos^2 x - \frac{3}{4} = 0 \Rightarrow \cos^2 x = \frac{3}{4}$

$\Rightarrow |\cos x| = \frac{\sqrt{3}}{2}$

$0 \leq x \leq \frac{\pi}{2} \Rightarrow \cos x = \frac{\sqrt{3}}{2}$

QUINDI $\boxed{x = \frac{\pi}{6}}$

$\boxed{f_{MIN} = \frac{7}{2}r^2}$

MAX ASSOLUTO: $\cos^2 x = 0 \rightarrow \cos x = 0 \rightarrow x = \frac{\pi}{2}$

MAX RELATIVO NON ASSOLUTO: $\cos^2 x = 1$

$\downarrow \quad 0 \leq x \leq \frac{\pi}{2}$

$\cos x = 1$

$\downarrow$

$x = 0.$