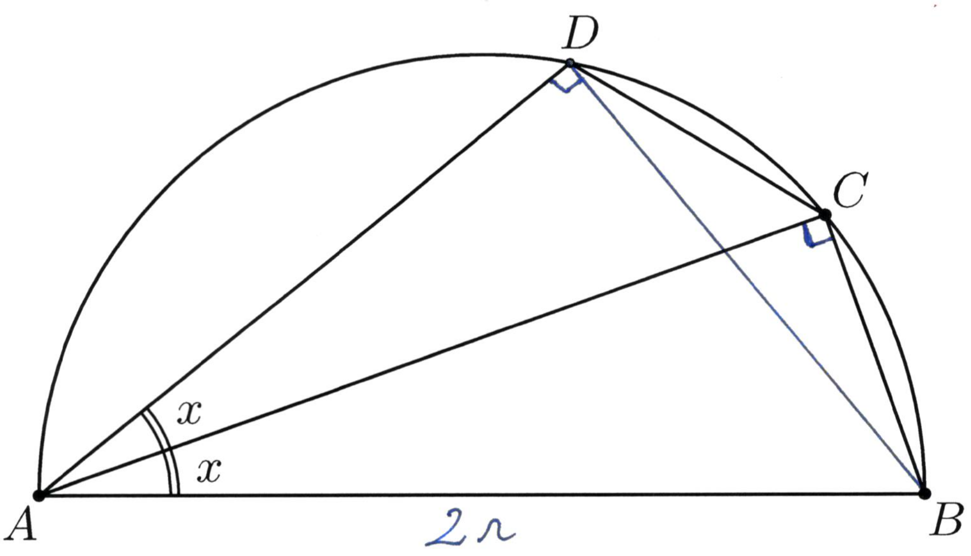
1)



DOMINIO

$$\boxed{0 \le x \le \frac{\pi}{4}}$$

$$\overline{AD} = 2r \cdot \cos(2x) = 2r \cdot (1 - 2\sin^2 x)$$

$$\overline{CB} = 2r \sin x$$

$$\begin{aligned}\overline{AD} + \overline{CB} &= 2r(1-2\sin^2 x) + 2r\sin x = \\ &= 2r(-2\sin^2 x + \sin x + 1) = \\ &= -4r\left(\sin^2 x - \frac{1}{2}\sin x - \frac{1}{2}\right) = \\ &= -4r\left[\left(\sin x - \frac{1}{4}\right)^2 - \frac{1}{16} - \frac{1}{2}\right] = \\ &= \frac{9}{4}r - 4r\left(\sin x - \frac{1}{4}\right)^2\end{aligned}$$

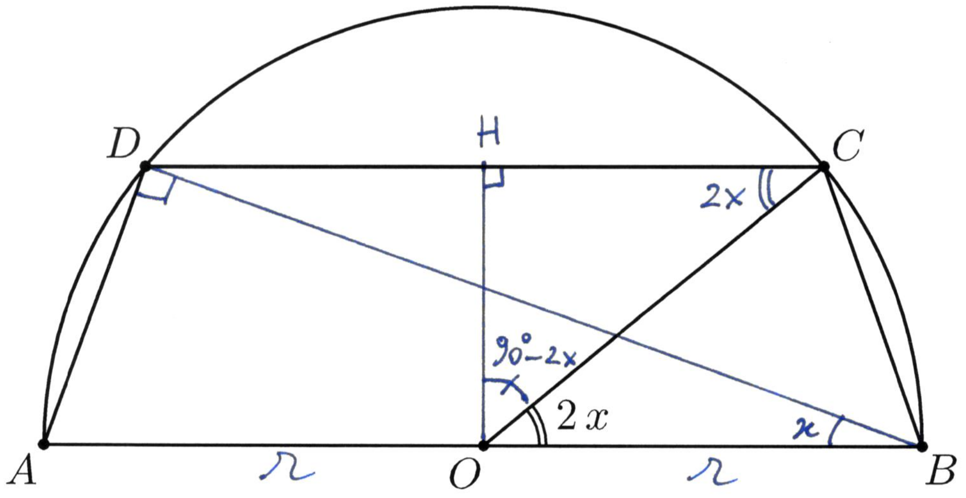
IL MAX SI OTTIENE QUANDO $\sin x - \frac{1}{4} = 0$

$\Rightarrow \sin x = \frac{1}{4} \Rightarrow x = \arcsin\frac{1}{4}$ ($\approx 14{,}48°$).

ACCETTABILE.

$$\boxed{y_{MAX} = \frac{9}{4}r}$$

2)



DOMINIO

$$\boxed{0 \le x \le \frac{\pi}{4}}$$

$$2p = \overline{AB} + 2 \cdot \overline{AD} + \overline{CD}$$

$\overline{AB} = 2r$

$\overline{AD} = 2r \sin x$

$\overline{CD} = 2 \cdot \overline{CH} = 2 \cdot r \cdot \cos(2x) = 2r(1 - 2\sin^2 x)$

$$2p(x) = 2r + 2 \cdot (2r \sin x) + 2r(1 - 2\sin^2 x) =$$

$$= 4r + 4r \sin x - 4r \sin^2 x =$$

$$= 4r + 4r(-\sin^2 x + \sin x) =$$

$$= 4r + 4r \cdot \left[-\left(\sin x - \frac{1}{2}\right)^2 + \frac{1}{4}\right] =$$

$$= 4r + r - 4r\left(\sin x - \frac{1}{2}\right)^2 =$$

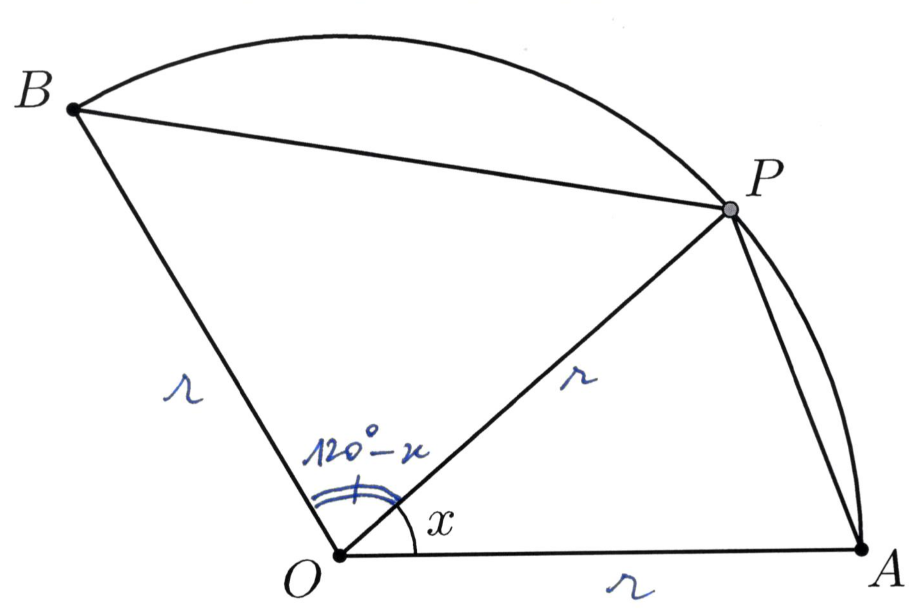
$$= 5r - 4r\left(\sin x - \frac{1}{2}\right)^2$$

IL MAX SI OTTIENE PER $\sin x - \frac{1}{2} = 0$

$\Rightarrow \quad x = \frac{\pi}{6} \quad (= 30°)$ → ABCD È LA <u>METÀ</u> DI UN <u>ESAGONO REGOLARE</u>

$$\boxed{y_{MAX} = 5r}$$

3)



DOMINIO

$$\boxed{0 \le x \le \frac{2}{3}\pi}$$

$$S(x) = S(OAP) + S(OPB) =$$

$$= \frac{1}{2} r \cdot r \cdot \sin x + \frac{1}{2} \cdot r \cdot r \cdot \sin(120° - x)$$

$$= \frac{1}{2} r^2 \left(\sin x + \sin(120° - x)\right).$$

$$\begin{cases} \sin(120° - x) = \underbrace{\sin(120°)}_{\frac{\sqrt{3}}{2}} \cdot \cos(-x) + \underbrace{\cos(120°)}_{\left(-\frac{1}{2}\right)} \cdot \sin(-x) \\ \qquad\qquad\qquad = \frac{\sqrt{3}}{2} \cdot \cos(x) + \left(-\frac{1}{2}\right) \cdot (-\sin x) \\ \sin(120° - x) = \frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin x \end{cases}$$

DUNQUE: $S(x) = \frac{1}{2} r^2 \cdot \left(\sin x + \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x\right)$

$$= \frac{1}{2} r^2 \cdot \left(\frac{3}{2} \sin x + \frac{\sqrt{3}}{2} \cos x\right) =$$

$$= \frac{r^2}{4} \cdot \left(3 \sin x + \sqrt{3} \cos x\right).$$

CON L'ANGOLO AGGIUNTO:

$$S(x) = \frac{r^2}{4} \cdot \sqrt{12} \cdot \left(\frac{3}{\sqrt{12}} \sin x + \frac{\sqrt{3}}{\sqrt{12}} \cos x\right) =$$

$$= \frac{r^2}{4} \cdot 2\sqrt{3} \cdot \left(\underbrace{\frac{\sqrt{3}}{2}}_{\cos\varphi} \sin x + \underbrace{\frac{1}{2}}_{\sin\varphi} \cos x\right) =$$

QUINDI $\begin{cases} \cos\varphi = \frac{\sqrt{3}}{2} \\ \sin\varphi = \frac{1}{2} \end{cases} \longrightarrow \varphi = \frac{\pi}{6} + 2k\pi \quad (k \in \mathbb{Z})$

$$S(x) = \underbrace{\frac{r^2}{4} \cdot 2\sqrt{3}}_{\frac{r^2}{2}\sqrt{3}} \cdot \sin\left(x + \frac{\pi}{6}\right)$$

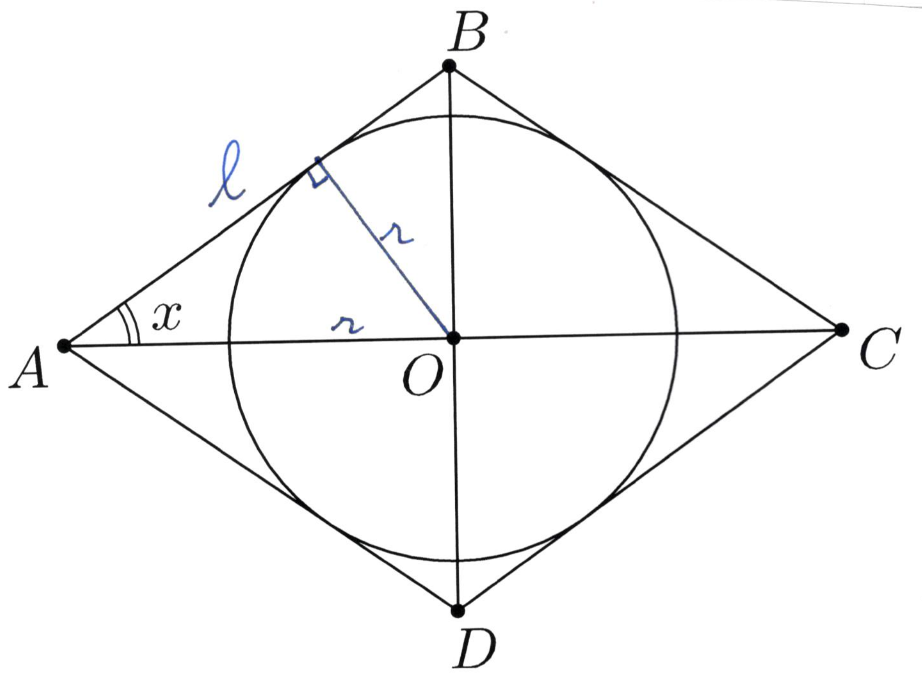
IL MAX SI OTTIENE QUANDO $x + \frac{\pi}{6} = \frac{\pi}{2} + 2k\pi$

$\Rightarrow x = \frac{\pi}{3} + 2k\pi$

è ACCETTABILE SOLO $\boxed{x = \frac{\pi}{3}}$ $(k=0)$

$$\boxed{S_{MAX} = \frac{\sqrt{3}}{2} r^2}$$

4)



DOMINIO

$$\boxed{0 < x < \frac{\pi}{2}}$$

$$\text{Area}(AOB) = \frac{1}{2} l \cdot r$$

(2° MODO PER) $\text{Area}(AOB) = \frac{1}{2}\, \overline{OA} \cdot \overline{OB}$

$\overline{OB} = l \cdot \sin x$

$\overline{OA} = l \cdot \cos x$

UGUAGLIANDO SI RICAVA:

$$\cancel{\tfrac{1}{2}}\, \cancel{l}\, r = \cancel{\tfrac{1}{2}}\, l^2 \sin x \cos x \Rightarrow$$

$$r = l \sin x \cos x \Rightarrow l = \frac{r}{\sin x \cdot \cos x}$$

OVVERO: $\boxed{l = \frac{2r}{\sin(2x)}}$

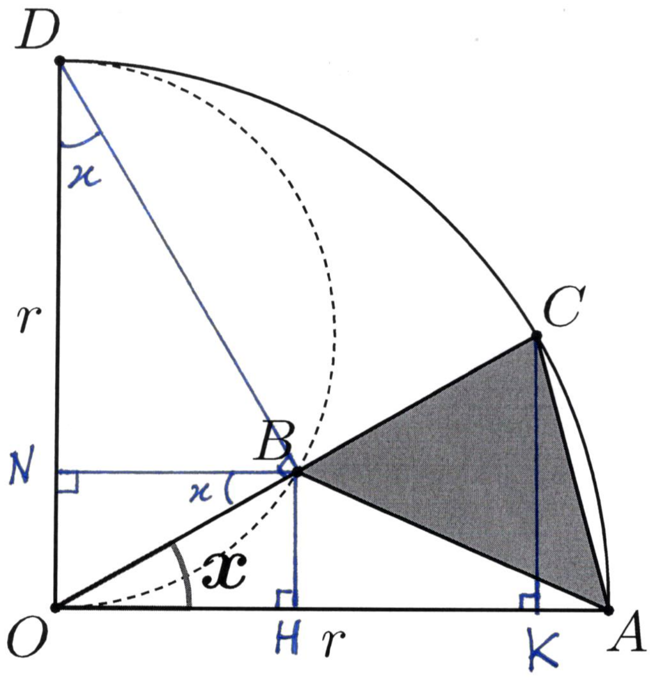
$$2p(x) = 4 \cdot l = 4 \cdot \frac{2r}{\sin(2x)} = \frac{8r}{\sin(2x)}$$

IL <u>MINIMO</u> SI OTTIENE QUANDO $\sin(2x)$ È <u>MASSIMO</u>

QUINDI $2x = \frac{\pi}{2} + 2k\pi \Longrightarrow x = \frac{\pi}{4} + k\pi$

ACCETT. SOLO CON $k=0$

5)



DOMINIO

$$\boxed{0 \le x \le \frac{\pi}{2}}$$

$$\overline{CK} = r \cdot \sin x$$

$$\overline{BH} = \overline{ON} = \overline{OB} \cdot \sin x \quad = r \sin^2 x$$

$$\downarrow$$

$$\overline{OB} = r \sin x$$

$$S_{ABC}(x) = S_{OAC}(x) - S_{OAB}(x) = \frac{1}{2} r \cdot (r \sin x) + $$
$$- \frac{1}{2} \cdot r \cdot (r \sin^2 x)$$

$$\Rightarrow S_{ABC}(x) = \frac{1}{2} r^2 (\sin x - \sin^2 x) =$$

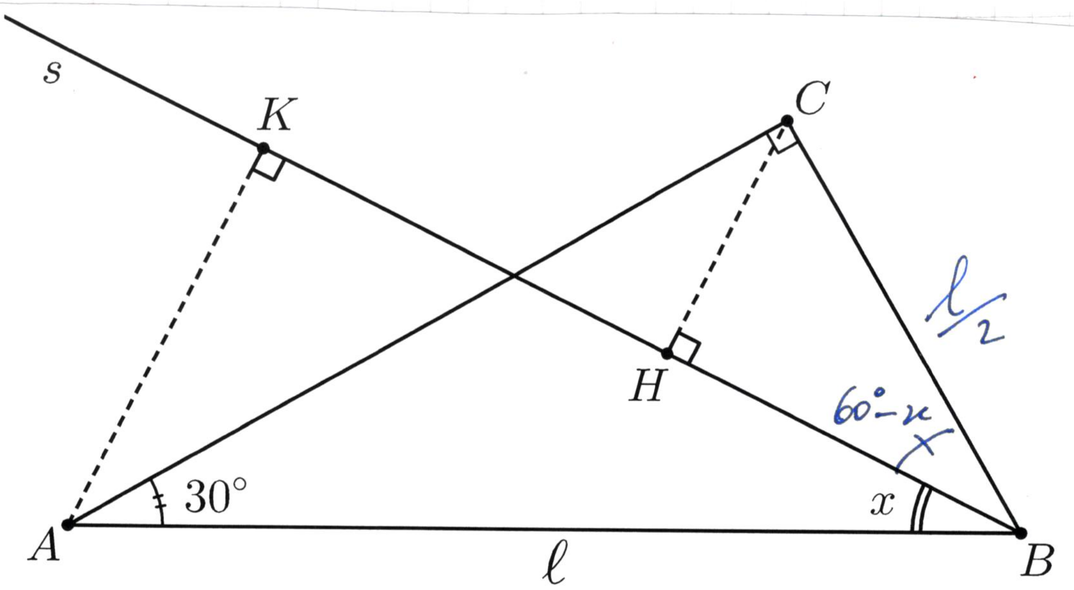
$$= \frac{1}{2} r^2 \cdot \left[-\left(\sin x - \frac{1}{2}\right)^2 + \frac{1}{4} \right] =$$

$$= \frac{1}{8} r^2 - \frac{1}{2} r^2 \left(\sin x - \frac{1}{2}\right)^2$$

IL MAX SI OTTIENE QUANDO $\sin x - \frac{1}{2} = 0$

$$\Rightarrow \boxed{x = \frac{\pi}{6}} \qquad \boxed{y_{MAX} = \frac{r^2}{8}}$$

6)



DOMINIO

$x \in \left[0; \frac{\pi}{3}\right]$

$\overline{BC} = \ell \cdot \sin(30°) = \ell/2.$

$\overline{AK} = \underbrace{\ell}_{\overline{AB}} \cdot \sin x \quad ; \quad \overline{CH} = \overline{CB} \cdot \sin(60° - x)$

$\overline{AK} + \overline{CH} = \ell \sin x + \frac{\ell}{2} \cdot \left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right) =$

$= \ell \cdot \left[\frac{\sqrt{3}}{4}\cos x + \frac{3}{4}\sin x\right] =$

$= \frac{\ell}{4} \cdot \left[3\sin x + \sqrt{3}\cos x\right] =$

$= \frac{\ell}{4} \cdot \sqrt{12} \cdot \left[\frac{3}{\sqrt{12}}\sin x + \frac{\sqrt{3}}{\sqrt{12}}\cos x\right] =$

$= \frac{\sqrt{3}}{2}\ell \cdot \left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right] = \ldots =$

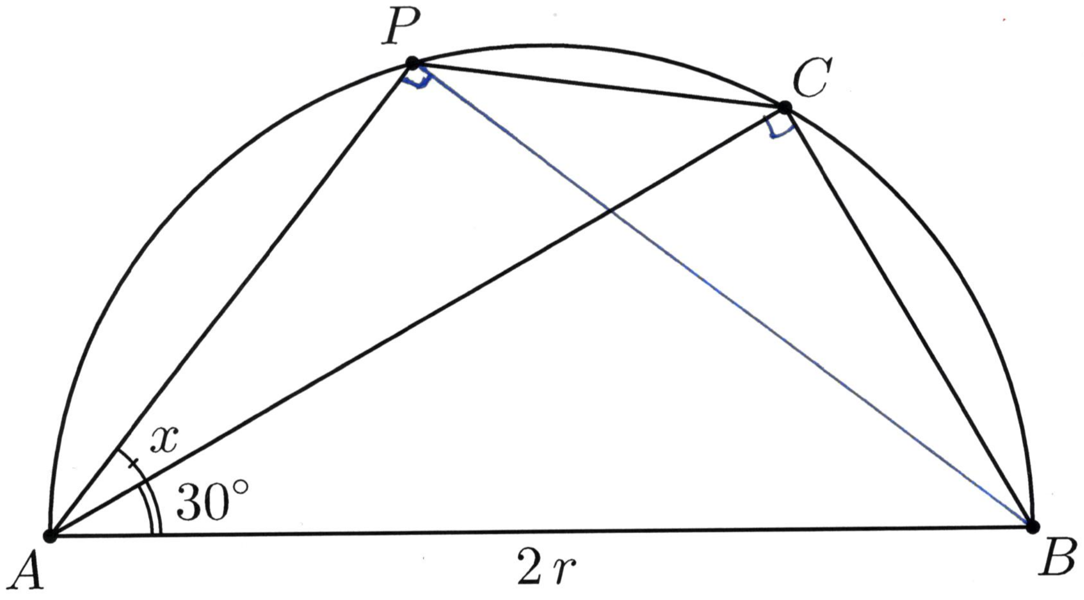
$= \frac{\sqrt{3}}{2} \cdot \ell \cdot \sin\left(x + \frac{\pi}{6}\right).$

IL MAX SI OTTIENE PER $x + \frac{\pi}{6} = \frac{\pi}{2} + 2k\pi$

$\Rightarrow x = \frac{\pi}{3} + 2k\pi$ SOLO $k=0$ È ACCETT.

$\boxed{x = \frac{\pi}{3}}$ $\boxed{y_{MAX} = \frac{\sqrt{3}}{2}\ell}$

7)



DOMINIO

$$\boxed{0 \leq x \leq \frac{\pi}{3}}$$

$$\text{Area}(APC) = \frac{1}{2} \cdot \overline{AP} \cdot \overline{AC} \cdot \sin x$$

$$\begin{cases} \overline{AP} = 2r \cdot \cos(30° + x) = 2r \cdot \left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right) \\ \overline{AC} = 2r \cdot \cos(30°) = \sqrt{3}\, r \end{cases}$$

$$S(x) = \frac{1}{2} \cdot \Big(\underbrace{r(\sqrt{3}\cos x - \sin x)}_{\overline{AP}}\Big) \cdot \underbrace{\sqrt{3}\, r}_{\overline{AC}} \cdot \sin x \Rightarrow$$

$$S(x) = \frac{\sqrt{3}}{2} r^2 \cdot \Big(\sqrt{3}\underbrace{\cos x \sin x}_{\frac{1}{2}\sin(2x)} - \underbrace{\sin^2 x}_{\frac{1-\cos(2x)}{2}}\Big)$$

QUINDI:

SOMMA = 1
QUADRATI

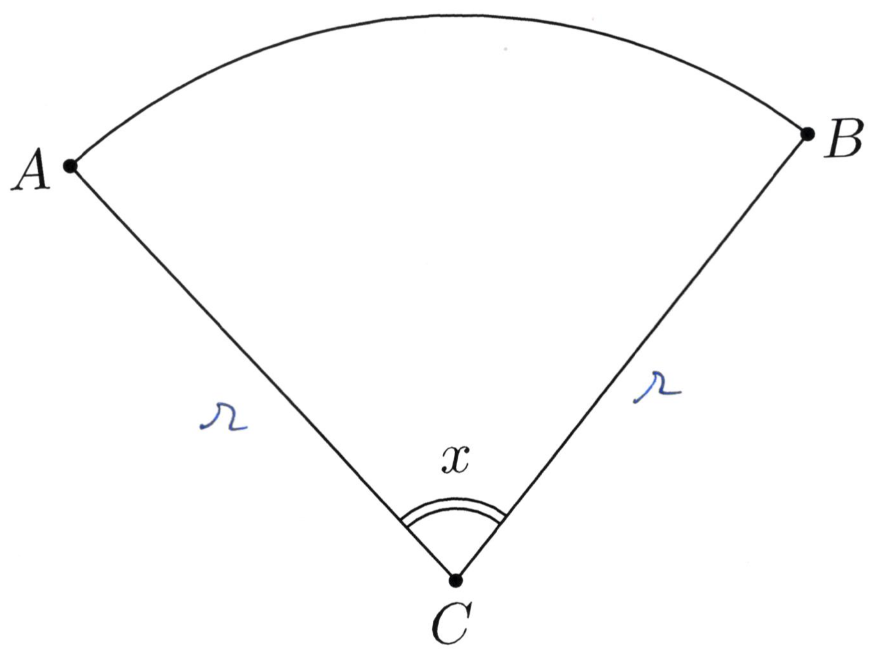
$$S(x) = \frac{\sqrt{3}}{2} r^2 \cdot \Big(\underbrace{\frac{\sqrt{3}}{2}}_{\cos\varphi}\sin(2x) + \underbrace{\frac{1}{2}}_{\sin\varphi}\cos(2x) - \frac{1}{2}\Big)$$

$$S(x) = \frac{\sqrt{3}}{2} r^2 \cdot \sin\left(2x + \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4} r^2 \qquad \left(S^*_{MAX} = \frac{\sqrt{3}}{4} r^2\right)$$

IL MAX SI OTTIENE PER $2x + \frac{\pi}{6} = \frac{\pi}{2} + 2k\pi \longrightarrow \boxed{x^* = \frac{\pi}{6}}$

(ACCETT. PER $k=0$)

8)



$$\boxed{0 < x < 2\pi}$$

$$S = \pi r^2 \cdot \frac{x}{2\pi} \rightarrow \boxed{S = \frac{1}{2} r^2 \cdot x}$$

S è ASSEGNATO, QUINDI:

$$r = \sqrt{\frac{2S}{x}}$$

$$f(x) = \underbrace{\overline{CA}}_{r} + \underbrace{\overline{CB}}_{r} + \underbrace{\overset{\frown}{AB}}_{x \cdot r} = 2r + x \cdot r = r(x+2)$$

$\downarrow$

CONTORNO
DA MINIMIZZARE

$$f(x) = r \cdot (x+2) = \sqrt{\frac{2S}{x}} \cdot (x+2) =$$

$$= \underbrace{\sqrt{2S}}_{\text{COSTANTE}} \cdot \frac{1}{\sqrt{x}} \cdot (x+2) = \sqrt{2S} \cdot \left(\sqrt{x} + \frac{2}{\sqrt{x}} \right) =$$

$$= \sqrt{2S} \cdot \left[\left(\sqrt[4]{x} - \frac{\sqrt{2}}{\sqrt[4]{x}} \right)^2 + 2\sqrt{2} \right];$$

IL MINIMO SI HA
PER $x = 2$ rad
($\approx 114{,}6°$)