12/03/21 – SOLUZIONI

1) $3x - y - 4 = 0 \longrightarrow y = 3x - 4 \quad (m = 3)$

$$y = 3(x+2) - 5 \Rightarrow \boxed{y = 3x + 1}$$

2) $3x - y - 4 \longrightarrow y = 3x - 4 \quad (m = 3)$

$$y = -\frac{1}{3}(x+2) - 5 \Rightarrow \boxed{y = -\frac{1}{3}x - \frac{17}{3}}$$

(CHE POSSIAMO RISCRIVERE NELLA FORMA
$3y = -x - 17 \longrightarrow x + 3y + 17 = 0$)

3) $$d(A, r) = \frac{|3 \cdot (-2) - (-5) - 4|}{\sqrt{3^2 + (-1)^2}} = \frac{|-5|}{\sqrt{10}} = \frac{5}{\sqrt{10}}$$

MOLTIPLICANDO PER $\sqrt{10}$ AL NUM. e AL DENOM. si OTTIENE:

$$d(A, r) = \frac{5}{\sqrt{10}} = \frac{5 \cdot \sqrt{10}}{\sqrt{10} \cdot \sqrt{10}} = \frac{5\sqrt{10}}{10} = \frac{\cancel{5} \cdot \sqrt{10}}{\cancel{10}_{2}} = \boxed{\frac{\sqrt{10}}{2}}$$

4) $$\begin{cases} y = 3x - 4 \\ y = -\frac{1}{3}x - \frac{17}{3} \end{cases} \Rightarrow \dots \begin{cases} x_H = -\frac{1}{2} \\ y_H = -\frac{11}{2} \end{cases} \Rightarrow \boxed{H\left(-\frac{1}{2}; -\frac{11}{2}\right)}$$

CONTROLLIAMO $d(A, r) = d(A, H)$

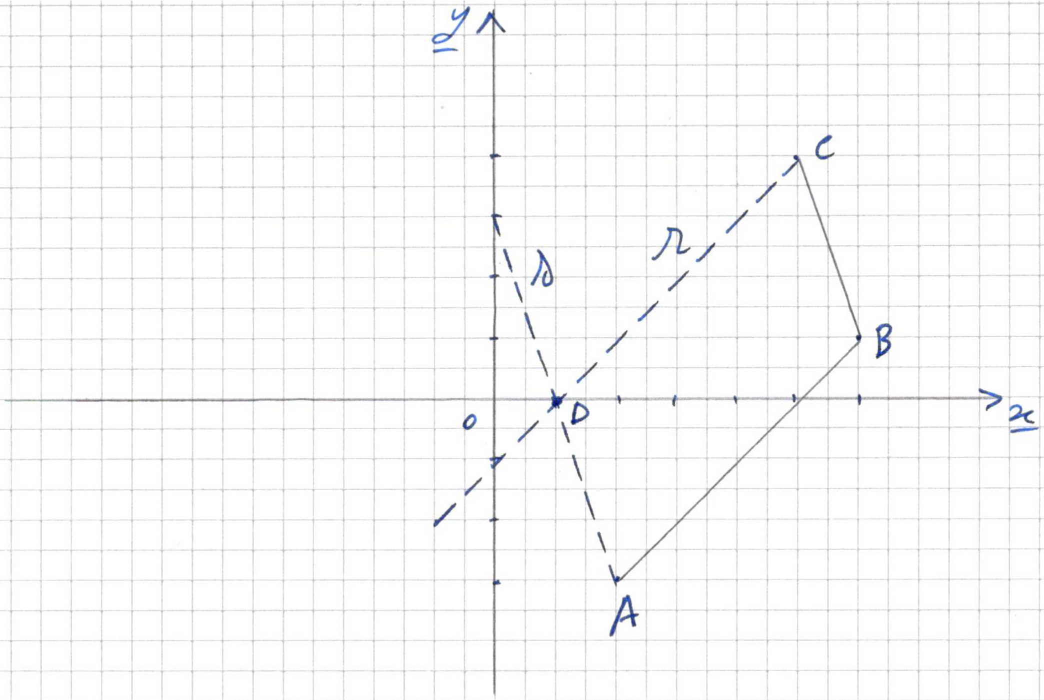
$$d(A, H) = \sqrt{\left(-\frac{1}{2} + 2\right)^2 + \left(-\frac{11}{2} + 5\right)^2} = \sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}}$$

MOLTIPLICO PER $\sqrt{2}$ AL NUM. e AL DENOM. si OTTIENE:

$$d(A, H) = \frac{\sqrt{5} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{10}}{2} \quad (\text{ok!})$$

stesso risultato
OTTENUTO ALL'es. 3.

5)



$$r: y = m_{AB} \cdot (x - x_C) + y_C$$

$$y = \frac{-3-1}{2-6} \cdot (x-5) + 4 \Rightarrow r: y = x - 1$$

$$s: y = m_{BC} \cdot (x - x_A) + y_A$$

$$y = \frac{4-1}{5-6} \cdot (x-2) - 3 \Rightarrow s: y = -3x + 3$$

$$D: \begin{cases} y = x - 1 \\ y = -3x + 3 \end{cases} \Rightarrow \dots \Rightarrow \begin{cases} x = 1 \\ y = 0 \end{cases} \Rightarrow \boxed{D(1;0)}$$

retta BC: $y = -3 \cdot (x-6) + 1 \Rightarrow y = -3x + 19$

retta AB: $y = 1 \cdot (x-6) + 1 \Rightarrow y = x - 5$
$(x - y - 5 = 0)$

$$\text{AREA} = \overline{AB} \cdot d(D; AB)$$

$$4 \cdot \sqrt{2} \cdot \frac{|1 - 0 - 5|}{\sqrt{1^2 + (-1)^2}} = 4\sqrt{2} \cdot \frac{4}{\sqrt{2}} = 16.$$

6) $C \in$ asse y ; $C \in r$

DOVE r è la retta passante per B e $\perp AB$.

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{1+2}{6-4} = \frac{3}{2}$$

$$r: y = -\frac{2}{3} \cdot (x-6) + 1 \rightarrow y = -\frac{2}{3}x + 5$$

$$C: \begin{cases} x = 0 \\ y = -\frac{2}{3}x + 5 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 5 \end{cases} \Rightarrow \boxed{C(0;5)}$$

retta CD: $y = \frac{3}{2} \cdot (x - x_C) + y_C$

$$y = \frac{3}{2}x + 5$$

retta AD: $y = -\frac{2}{3} \cdot (x - x_A) + y_A$

$$y = -\frac{2}{3} \cdot (x-4) - 2$$

$$y = -\frac{2}{3}x + \frac{2}{3}$$

$$D: \begin{cases} y = \frac{3}{2}x + 5 \\ y = -\frac{2}{3}x + \frac{2}{3} \end{cases} \Rightarrow \ldots \Rightarrow \begin{cases} x = -2 \\ y = 2 \end{cases} \Rightarrow \boxed{D(-2;2)}$$

$$\text{AREA} = \overline{AB} \cdot \overline{BC}$$

$$\sqrt{13} \cdot (2\sqrt{13}) = 2 \cdot 13 = \boxed{26}$$

N.B. $\overline{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} =$

$$= \sqrt{(2-4)^2 + (1+2)^2} = \sqrt{13}.$$

7) 1° METODO:

asse del SEGMENTO AB:

$$(x-4)^2+(y+2)^2=(x-2)^2+(y-1)^2$$

$$\cancel{x^2}+16-8x+\cancel{y^2}+\cancel{4}+4y=\cancel{x^2}+\cancel{4}-4x+\cancel{y^2}+1-2y$$

$$4x-6y-15=0$$

INTERSECANDO CON L'ASSE x SI TROVA $\boxed{C=\left(\frac{15}{4};0\right)}$

2° METODO: $C(t;0)$

$$\overline{AC}=\overline{BC} \Rightarrow \overline{AC}^2=\overline{BC}^2 \Rightarrow$$

$$(4-t)^2+(-2-0)^2=(2-t)^2+(1-0)^2 \Rightarrow$$

$$16+\cancel{t^2}-8t+\cancel{4}=\cancel{4}+\cancel{t^2}-4t+1$$

$$4t=15 \Rightarrow t=\frac{15}{4} \Rightarrow C\left(\frac{15}{4};0\right)$$

8) ANCHE QUI POSSIAMO PROCEDERE IN 2 MODI.

1° METODO: $\begin{cases} \text{asse}_{AB} \\ \text{asse } y \end{cases} \Rightarrow \begin{cases} 4x-6y-15=0 \\ x=0 \end{cases}$

$$\Rightarrow \begin{cases} y=-\frac{15}{6}=-\frac{5}{2} \\ x=0 \end{cases} \Rightarrow \boxed{D\left(0;-\frac{5}{2}\right)}$$

2° METODO: $D(0;t)$ $\quad \overline{AD}^2=\overline{BD}^2 \Rightarrow$

$$(4-0)^2+(-2-t)^2=(2-0)^2+(1-t)^2 \Rightarrow$$

$$16+\cancel{4}+\cancel{t^2}+4t=\cancel{4}+1+\cancel{t^2}-2t \Rightarrow 6t=-15$$

$$\Rightarrow t=-15/6=-\frac{5}{2} \Rightarrow D\left(0;-\frac{5}{2}\right)$$

9) $P(t;0)$

$$\overline{PB} = \frac{1}{2}\cdot\overline{PA} \Rightarrow \overline{PB}^2 = \frac{1}{4}\cdot\overline{PA}^2$$

$$\Rightarrow 4\cdot\overline{PB}^2 = \overline{PA}^2$$

$$4\cdot\left[(t-2)^2+(0-1)^2\right] = (t-4)^2+(0+2)^2$$

$$\Rightarrow 4t^2+\cancel{16}-16t+\cancel{4} = t^2+\cancel{16}-8t+\cancel{4}$$

$$\Rightarrow 3t^2-8t=0 \Rightarrow t(\underbrace{3t-8}_{t_2=8/3})=0$$

$t_1=0$

i punti richiesti sono:

$$\boxed{P_1(0;0) \text{ e } P_2\left(\frac{8}{3};0\right)}$$

10) $P(0;t)$

$$5\cdot\overline{PB} = 2\cdot\overline{PA} \Rightarrow 25\cdot\overline{PB}^2 = 4\cdot\overline{PA}^2$$

$$25\cdot\left[(0-2)^2+(t-1)^2\right] = 4\cdot\left[(0-4)^2+(t+2)^2\right]$$

$$\Rightarrow 100+25t^2+25-50t = 64+4t^2+16+16t$$

$$\Rightarrow 21t^2-66t+45=0$$

divido per 3: $\quad 7t^2-22t+15=0$

$$\Delta = (-22)^2-4\cdot7\cdot15 = 64 = 8^2$$

$$t_{1,2} = \frac{22\pm8}{2\cdot7} = \frac{22\pm8}{14} = \begin{cases}15/7\\1\end{cases}$$

$$\boxed{P_1\left(0;\frac{15}{7}\right);\quad P_2(0;1)}$$