SOLUZIONI 6/12/2021

1) $$\begin{pmatrix}1\\m\end{pmatrix}\cdot\begin{pmatrix}1\\2\end{pmatrix}=\sqrt{1+m^2}\cdot\sqrt{5}\cdot\frac{4}{5}\cdot(\pm 1)$$

$$(1\cdot 1+m\cdot 2)^2=\left(\sqrt{5+5m^2}\cdot\frac{4}{5}\right)^2\Rightarrow$$

$$4m^2+20m-11=0$$

$$m_{1,2}= \begin{cases} \nearrow \frac{1}{2} \Rightarrow \boxed{y=\frac{1}{2}x} \\ \searrow -\frac{11}{2} \Rightarrow \boxed{y=-\frac{11}{2}x}\end{cases}$$

2° METODO: $$\frac{m-2}{1+m\cdot 2}=\tan\Big(\underbrace{\arccos\left(\tfrac{4}{5}\right)}_{\alpha}\Big)\cdot(\pm 1)$$

$$\alpha=\arccos\left(\frac{4}{5}\right)\rightarrow\cos\alpha=\frac{4}{5}\rightarrow\sin\alpha=3/5$$

QUINDI $$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{3/5}{4/5}=\frac{3}{5}\cdot\frac{5}{4}=\frac{3}{4}$$

$$\frac{m-2}{1+2m}=\pm\frac{3}{4}\Rightarrow m_{1,2}= \begin{cases}\nearrow -11/2\\ \searrow \frac{1}{2}\end{cases}$$

3° METODO: RUOTO LA RETTA $r: y=2x$

IN SENSO ORARIO CON $\cos\alpha=4/5$:

$$\begin{cases}x'=4/5\,x+3/5\,y\\ y'=-3/5\,x+4/5\,y\end{cases}\longrightarrow\begin{cases}x=4/5\,x'-\frac{3}{5}y'\\ y=3/5\,x'+\frac{4}{5}x'\end{cases}$$

$$y=2x$$

$$\downarrow$$

$$\frac{3}{5}x'+\frac{4}{5}y'=2\cdot\left(\frac{4}{5}x'-\frac{3}{5}y'\right)\Rightarrow\boxed{y'=\frac{1}{2}x'}$$

RUOTIAMO ORA LA RETTA $r: y=2x$

<u>IN SENSO ANTIORARIO</u> CON $\cos\alpha = 4/5$:

$$\begin{cases} x' = \frac{4}{5}x - \frac{3}{5}y \\ y' = \frac{3}{5}x + \frac{4}{5}y \end{cases} \to \begin{cases} x = \frac{4}{5}x' + \frac{3}{5}y' \\ y = -\frac{3}{5}x' + \frac{4}{5}y' \end{cases}$$

$$y = 2x$$

$$-\frac{3}{5}x' + \frac{4}{5}y' = 2\cdot\left(\frac{4}{5}x' + \frac{3}{5}y'\right) \Rightarrow \boxed{y' = -\frac{11}{2}x'}$$

2)
$$\begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ m \end{pmatrix} = \sqrt{1^2+3^2} \cdot \sqrt{1+m^2} \cdot \cos(45^\circ) \cdot (\pm 1)$$

$$3 + m = \sqrt{10 + 10m^2} \cdot \frac{1}{\sqrt{2}} \cdot (\pm 1)$$

$$(3+m)^2 = (10 + 10m^2) \cdot \frac{1}{2} \Rightarrow m_{1,2} = \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$$

QUINDI le due rette sono:

$y = 2x$; $y = -\frac{1}{2}x$.

<u>2° METODO</u>:
$$\frac{m - \frac{1}{3}}{1 + m\cdot\frac{1}{3}} = \pm \tan(45^\circ)$$

$$\frac{m - \frac{1}{3}}{1 + \frac{m}{3}} = \pm 1 \Rightarrow m_{1,2} = \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$$

<u>3° METODO</u>: CONSIDERO LA ROTAZ. DI ANGOLO 45° IN SENSO <u>ANTIORARIO</u>

$$\begin{cases} x' = \frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}y \\ y' = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \end{cases} \Rightarrow \begin{cases} x = \frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y' \\ y = -\frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y' \end{cases}$$

$$x - 3y = 0$$

$$\frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y' - 3\cdot\left(-\frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y'\right) = 0$$

$$\Rightarrow \boxed{y' = 2x'}.$$

VEDIAMO ORA LA ROTAZIONE DI 45° IN SENSO ORARIO:

$$\begin{cases} x' = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \\ y' = -\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \end{cases} \longrightarrow \begin{cases} x = \frac{1}{\sqrt{2}}x' - \frac{1}{\sqrt{2}}y' \\ y = \frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y' \end{cases}$$

$$x - 3y = 0$$

$$\frac{1}{\sqrt{2}}x' - \frac{1}{\sqrt{2}}y' - 3\cdot\left(\frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y'\right) = 0$$

$$\Rightarrow \boxed{y' = -\frac{1}{2}x'}.$$

3) $A(5;3)$, $B(2;1)$ $\rightarrow$ $\underbrace{y = \frac{2}{3}x - \frac{1}{3}}_{\text{retta}_{AB}}$

VEDIAMO LA RETTA CON PENDENZA NEGATIVA.

1° METODO: $\begin{pmatrix} 1 \\ m \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \sqrt{1+m^2}\cdot\sqrt{13}\cdot\underbrace{\cos(60°)}_{\frac{1}{2}}$

$$\Rightarrow \quad -3 - 2m = \sqrt{13 + 13m^2}\cdot\frac{1}{2}$$

$$\Rightarrow \quad (-3-2m)^2 = (13 + 13m^2)\cdot\frac{1}{4}$$

$$m_{1,2} = -8 \pm \frac{13}{\sqrt{3}}$$

LA SOLUZ. ACCETTABILE È

$$m = -8 - \frac{13}{\sqrt{3}}$$

QUINDI la 2ª retta è:

$$\boxed{y=\left(-8-\frac{13}{\sqrt{3}}\right)(x-5)+3}.$$

2° METODO: $\dfrac{m-\frac{2}{3}}{1+m\cdot\frac{2}{3}}=\underbrace{\tan(60°)}_{\sqrt{3}}$

$$\Rightarrow m=-8-\frac{13}{\sqrt{3}}$$

3° METODO: CONSIDERO LA ROTAZIONE di 60° IN SENSO ANTIORARIO:

$$\begin{cases} x'=\frac{1}{2}x-\frac{\sqrt{3}}{2}y \\ y'=\frac{\sqrt{3}}{2}x+\frac{1}{2}y \end{cases} \longrightarrow \begin{cases} x=\frac{1}{2}x'+\frac{\sqrt{3}}{2}y' \\ y=-\frac{\sqrt{3}}{2}x'+\frac{1}{2}y' \end{cases}$$

$y=\frac{2}{3}x$ (BASTA CONOSCERE LA NUOVA PENDENZA)

$$-\frac{\sqrt{3}}{2}x'+\frac{1}{2}y'=\frac{2}{3}\cdot\left(\frac{1}{2}x'+\frac{\sqrt{3}}{2}y'\right) \Rightarrow \dots \Rightarrow$$

$$y'=\frac{2+3\sqrt{3}}{3-2\sqrt{3}}x' \Rightarrow y'=\left(-8-\frac{13}{\sqrt{3}}\right)x'$$

PER CONCLUDERE BASTA SCRIVERE LA PARALLELA PASSANTE PER IL PUNTO $A(5;3)$.

4) $y=m(x+2)+1 \rightarrow mx-y+1+2m=0$

$$\frac{|m\cdot 3-5+1+2m|}{\sqrt{m^2+1}}=\frac{6}{\sqrt{5}} \Rightarrow \frac{|5m-4|}{\sqrt{m^2+1}}=\frac{6}{\sqrt{5}}$$

A QUESTO PUNTO ELEVIAMO AL QUADRATO:

$$(5m-4)^2 = (m^2+1)\cdot\left(\frac{6}{\sqrt{5}}\right)^2 \Rightarrow$$

$$89m^2 - 200m + 44 = 0$$

$$m_{1,2} = \begin{cases} 2 \\ 22/89 \quad (\approx 0{,}2472) \end{cases}$$

$$\begin{pmatrix}1\\2\end{pmatrix}\cdot\begin{pmatrix}89\\22\end{pmatrix} = \sqrt{1^2+2^2}\cdot\sqrt{89^2+22^2}\cdot\cos\alpha$$

$$1\cdot 89 + 2\cdot 22 = \sqrt{5}\cdot(41\cdot\sqrt{5})\cdot\cos\alpha$$

$$\cos\alpha = \frac{133}{\sqrt{5}\cdot 41\cdot\sqrt{5}} \Rightarrow \cos\alpha = \frac{133}{205}$$

$$\boxed{\alpha = \arccos\left(\frac{133}{205}\right) \approx 49{,}55°}$$

<u>2° METODO:</u> $\alpha = 2\cdot\beta$;

$$\sin\beta = \frac{r}{\overline{CP}} = \frac{6/\sqrt{5}}{\sqrt{41}} = \frac{6}{\sqrt{205}}$$

$$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{36}{205}} = \frac{13}{\sqrt{205}}$$

$$\cos\alpha = \cos(2\beta) = 2\cdot\cos^2\beta - 1 = 2\cdot\left(\frac{13}{\sqrt{205}}\right)^2 - 1 \Rightarrow$$

$$\cos\alpha = 2\cdot\frac{169}{205} - 1 \Rightarrow \cos\alpha = \frac{133}{205}$$

$$\boxed{\alpha = \arccos\left(\frac{133}{205}\right) \approx 49{,}55°}.$$