SOLUZ. 11/12/21

1) $\begin{cases} x+y+k=0 \\ 2x+y-6=0 \end{cases} \rightarrow \begin{cases} x=k+6 \\ y=-2k-6 \end{cases}$

$P(k+6;-2k-6) \qquad \overline{AP}=\sqrt{5} \rightarrow \overline{AP}^2=5$

$(k+6-3)^2+(-2k-6)^2=5 \Rightarrow \ldots \Rightarrow 5k^2+30k+40=0$

$\boxed{k_1=-2 \; ; \; k_2=-4}$

2)

$2kx-x-ky+3y+4-3k=0$

$k(2x-y-3)-x+3y+4=0$

GENERATRICI: $2x-y-3=0 \; ; \; x-3y-4=0$

$C: \begin{cases} 2x-y-3=0 \\ x-3y-4=0 \end{cases} \Rightarrow C(1;-1).$

$$\frac{|(2k-1)\cdot(-1)-(k-3)\cdot 2+4-3k|}{\sqrt{(2k-1)^2+(k-3)^2}}=\frac{4}{\sqrt{5}}$$

$$\frac{|11-7k|}{\sqrt{5k^2-10k+10}}=\frac{4}{\sqrt{5}} \Rightarrow \frac{(11-7k)^2}{5k^2-10k+10}=\frac{16}{5}$$

$\Rightarrow \boxed{k_1=1 \; ; \; k_2=\frac{89}{33}}$

RETTA $\perp$ $3x-2y+1=0 \Rightarrow \begin{pmatrix} 2k-1 \\ 3-k \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \end{pmatrix}=0$

$\Rightarrow (2k-1)\cdot 3+(3-k)\cdot(-2)=0 \Rightarrow \boxed{k=\frac{9}{8}}$

RETTA $//$ $4x-5y-30=0 \Rightarrow \begin{pmatrix} 2k-1 \\ 3-k \end{pmatrix} // \begin{pmatrix} 4 \\ -5 \end{pmatrix} \Rightarrow \ldots$

$(2k-1)\cdot(-5)-(3-k)\cdot 4=0 \Rightarrow \boxed{k=-\frac{7}{6}}$

3) $r: y=\frac{2}{5}(x-4)-5 \Rightarrow y=\frac{2}{5}x-\frac{33}{5}$

$asse_{AC}: (x-3)^2+(y+2)^2=(x-7)^2+(y-0)^2$

$\Rightarrow y=9-2x$

$B: \begin{cases} y=9-2x \\ y=\frac{2}{5}x-\frac{33}{5} \end{cases} \Rightarrow \boxed{B\left(\frac{13}{2};-4\right)}$

$M_{AC}=\left(\frac{3+7}{2};\frac{-2+0}{2}\right)=(5;-1).$

D è il simmetrico di B rispetto a M_{AC}:

$\begin{cases} x_D=2\cdot 5-\frac{13}{2}=\frac{7}{2} \\ y_D=2\cdot(-1)-(-4)=2 \end{cases} \Rightarrow \boxed{D\left(\frac{7}{2};2\right)}$

$AREA_{ROMBO}=\frac{1}{2}\cdot d_{AC}\cdot d_{BD}=\frac{1}{2}\cdot 2\sqrt{5}\cdot 3\sqrt{5}=15.$

4) $r: 4x+3y-6=0$; $s: x-3=0.$

$\frac{|4x+3y-6|}{\sqrt{4^2+3^2}}=2\cdot|x-3| \Rightarrow$

$\frac{(4x+3y-6)^2}{25}=4\cdot(x-3)^2 \Rightarrow$

in questo modo troviamo

$34x^2-24xy-9y^2-252x+36y+414=0$

risolvendo rispetto a y troviamo le due rette.

2° METODO (PIÙ VELOCE)

$$|4x+3y-6| = 5 \cdot 2 \cdot |x-3|$$

$$4x+3y-6 = \pm 10(x-3)$$

$4x+3y-6 = 10x-30$ | $4x+3y-6 = -10x+30$

$\boxed{2x-y-8=0}$ | $\boxed{14x+3y-36=0}$

IL LUOGO $\mathcal{L}$ è COSTITUITO DALL'UNIONE delle 2 RETTE.

5) $r: x+2y-1=0$; $s: x-2y+4=0$

bisettrici: $$\frac{|x+2y-1|}{\cancel{\sqrt{1^2+2^2}}} = \frac{|x-2y+4|}{\cancel{\sqrt{1^2+2^2}}} \Rightarrow$$

$$|x+2y-1| = |x-2y+4|$$

$$x+2y-1 = \pm(x-2y+4)$$

$x+2y-1 = x-2y+4$ | $x+2y-1 = -x+2y-4$

$y = \frac{5}{4}$ | $x = -\frac{3}{2}$

DETERMINIAMO ORA L'ASSE del SEGMENTO AB:

$$(x-3)^2+(y-2)^2 = (x+1)^2+(y-0)^2 \Rightarrow \dots \Rightarrow$$

$$y = 3-2x$$

$$P_1: \begin{cases} y = \frac{5}{4} \\ y = 3-2x \end{cases} \Rightarrow \boxed{P_1\left(\frac{7}{8}; \frac{5}{4}\right)}$$

$$P_2: \begin{cases} x = -3/2 \\ y = 3-2x \end{cases} \Rightarrow \boxed{P_2\left(-\frac{3}{2}; 6\right)}.$$

2ª PARTE: $u: x+2y+h=0$

$$d(r;u)=\frac{3}{\sqrt{5}} \Rightarrow d((1;0);u)=\frac{3}{\sqrt{5}} \Rightarrow$$

$$\frac{|1+2\cdot 0+h|}{\cancel{\sqrt{5}}}=\frac{3}{\cancel{\sqrt{5}}} \Rightarrow |1+h|=3 \Rightarrow h_{1,2}=\begin{cases}2\\-4\end{cases}$$

$x+2y+2=0 \rightarrow y=-\frac{1}{2}x-1$ ok

$x+2y-4=0 \rightarrow y=-\frac{1}{2}x+2$ NO

QUINDI $u: y=-\frac{1}{2}x-1$; $u: x=-2y-2$.

$Q(-2t-2;t)$

$$d(Q,A)=\sqrt{\frac{17}{13}}\cdot d(Q,B) \Rightarrow d^2(Q,A)=\frac{17}{13}\cdot d^2(Q,B)$$

$$(-2t-2-3)^2+(t-2)^2=\frac{17}{13}\cdot\left[(-2t-2+1)^2+(t-0)^2\right]$$

$$\Rightarrow \ldots \Rightarrow t^2-7t-18=0 \Rightarrow t_{1,2}=\begin{cases}9\\-2\end{cases}$$

$\boxed{Q_1(-20;9)\ ;\ Q_2(2;-2)}$.

6) $r: y=\frac{4-1}{0-4}\cdot(x-0)+4 \Rightarrow y=-\frac{3}{4}x+4$

$(3x+4y=16)$

$s: 3x+4y+h=0$

$$d(r;s)=2 \Rightarrow d(A,s)=2 \Rightarrow \frac{|3\cdot 0+4\cdot 4+h|}{\sqrt{3^2+4^2}}=2$$

$$\Rightarrow |16+h|=5\cdot 2 \Rightarrow h_{1,2}=\begin{cases}-6\\-26\end{cases}$$

$3x+4y-6=0 \rightarrow y=-\frac{3}{4}x+\frac{3}{2}$ con $\frac{3}{2}<4 \rightarrow$ ok!

$3x+4y-26=0 \rightarrow y=-\frac{3}{4}x+\frac{13}{2}$ con $\frac{13}{2}>4 \rightarrow$ NO!

QUINDI $s: 3x+4y-6=0$

$u: y = mx+3$, CON $m>0$.

$$\begin{pmatrix}1\\m\end{pmatrix}\cdot\begin{pmatrix}4\\-3\end{pmatrix} = \sqrt{1+m^2}\cdot\sqrt{4^2+3^2}\cdot\underbrace{\cos\delta}_{33/65} \Longrightarrow$$

$$1\cdot 4 + m\cdot(-3) = \sqrt{1+m^2}\cdot 5\cdot\frac{33}{65} \Longrightarrow$$

$$4-3m = \frac{33}{13}\cdot\sqrt{1+m^2} \Longrightarrow$$

$$(4-3m)^2 = \left(\frac{33}{13}\right)^2\cdot(1+m^2) \Longrightarrow$$

$$432m^2 - 4'056m + 1'615 = 0$$

$$m_{1,2} = \begin{cases} 5/12 \quad (\approx 0{,}417) \\ \dfrac{323}{36} \quad (\approx 8{,}972)\end{cases}$$

LA SOLUZIONE ACCETTABILE È $m = \frac{5}{12}$

INFATTI CON L'ALTRA PENDENZA $\left(m_2 = \frac{323}{36}\right)$

OTTENIAMO UN ANGOLO δ OTTUSO.

IN DEFINITIVA:

$$\boxed{R: \begin{cases} 3x+4y \le 16 \\ 3x+4y \ge 6 \\ 5x-12y \ge -36 \end{cases}}$$