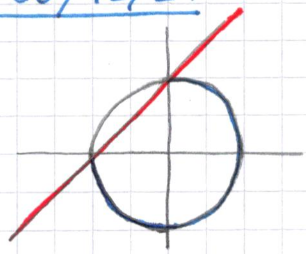
SOLUZ. 20/12/21

1) $\cos x - \sin x > -1$

$X - Y > -1$

$Y < X + 1$



$-\pi < x < \frac{\pi}{2}$ in $[-\pi; \pi]$

SOLUZIONI: $\boxed{-\pi + 2k\pi < x < \frac{\pi}{2} + 2k\pi}$

2) $\begin{cases} x = x - y + 3 \\ y = -y + 6 \end{cases} \Rightarrow \begin{cases} y = 3 \\ 2y = 6 \end{cases} \Rightarrow y = 3$

RETTA DI PUNTI FISSI

RETTE INVARIANTI:

$y' = mx' + k \Rightarrow -y + 6 = m \cdot (x - y + 3) + k \Rightarrow$

$-y + 6 = mx - my + 3m + k \Rightarrow y = \frac{m}{m-1} x + \frac{k + 3m - 6}{m-1}$

$\begin{cases} \frac{m}{m-1} = m \\ \frac{k + 3m - 6}{m-1} = k \end{cases} \Rightarrow \begin{cases} m = 0 \vee m = 2 \\ k + 3m - 6 = k(m-1) \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} m = 0 \\ k - 6 = -k \end{cases} \vee \begin{cases} m = 2 \\ k + \cancel{3 \cdot 2} - \cancel{6} = k \cdot (2-1) \end{cases}$

$\begin{cases} m = 0 \\ k = 3 \end{cases} \vee \begin{cases} m = 2 \\ k = k \end{cases}$

$\boxed{y = 3}$ $\qquad$ $\boxed{y = 2x + k}$

LO SAPEVAMO GIÀ! $\qquad$ INFINITE RETTE INVARIANTI

$\gamma: x^2+y^2=1$

$$\begin{cases} x' = x-(6-y')+3 \\ y = 6-y' \end{cases} \Rightarrow \begin{cases} x = x'-y'+3 \\ y = 6-y' \end{cases}$$

$(\varphi = \varphi^{-1})$

$\gamma': (x'-y'+3)^2+(6-y')^2=1$

$\gamma': x'^2+y'^2+9-2x'y'+6x'-6y'+$
$\quad +36+y'^2-12y'-1=0$

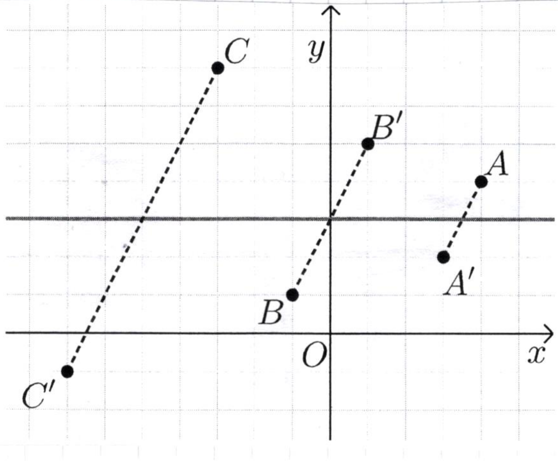
$\boxed{x'^2+2y'^2-2x'y'+6x'-18y'+44=0}$ ELLISSE

$\text{AREA}(\gamma') = \underbrace{|\det A|}\cdot\underbrace{\text{AREA}(\gamma)}$

$\text{AREA}(\gamma') = |-1| \cdot \pi\cdot 1^2$

$\boxed{\text{AREA}(\gamma') = \pi}$

φ è LA SIMMETRIA OBLIQUA RISPETTO ALLA RETTA $y=3$, PARALLELA ALLA RETTA $y=2x$. INFATTI, PRESO UN PUNTO $P(x_0;y_0)$, L'IMMAGINE SI OTTIENE INTERSECANDO LA RETTA $r_P \parallel y=2x$ E PASSANTE PER P CON LA RETTA $s: y=3$, SI DETERMINA IL PUNTO M ED INFINE SI CALCOLA IL SIMMETRICO P' di P RISP. A M.



3) $\overrightarrow{C_1A} = \begin{pmatrix} 4-2 \\ 4-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$; $\overrightarrow{C_1A'} = \begin{pmatrix} 5-2 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$$\underbrace{\overrightarrow{C_1A} \cdot \overrightarrow{C_1A'}} = \|\overrightarrow{C_1A}\| \cdot \|\overrightarrow{C_1A'}\| \cdot \cos\alpha_1$$

$$2 \cdot 3 + 3 \cdot 2 = \sqrt{13} \cdot \sqrt{13} \cdot \cos\alpha_1$$

$$\cos\alpha_1 = \frac{12}{13} ; \qquad \sqrt{1 - \left(\frac{12}{13}\right)^2} = \frac{5}{13}$$

$$\begin{cases} x' = \frac{12}{13}(x-2) + \frac{5}{13}(y-1) + 2 \\ y' = -\frac{5}{13}(x-2) + \frac{12}{13}(y-1) + 1 \end{cases}$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x'-0 \\ y'-0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x'' = y' \\ y'' = -x' \end{cases}$$

$$\psi : \begin{cases} x'' = -\frac{5}{13}(x-2) + \frac{12}{13}(y-1) + 1 \\ y'' = -\frac{12}{13}(x-2) - \frac{5}{13}(y-1) - 2 \end{cases}$$

$$\psi : \begin{cases} x'' = -\frac{5}{13}x + \frac{12}{13}y + \frac{11}{13} \\ y'' = -\frac{12}{13}x - \frac{5}{13}y + \frac{3}{13} \end{cases}$$

è UNA ROTAZIONE, di ANGOLO $\alpha = \arccos\left(-\frac{5}{13}\right)$ SENSO ORARIO ($\approx 112{,}62°$)

CENTRO di ψ:

$$\begin{cases} x = -\frac{5}{13}x + \frac{12}{13}y + \frac{11}{13} \\ y = -\frac{12}{13}x - \frac{5}{13}y + \frac{3}{13} \end{cases} \Longrightarrow$$

$$\Longrightarrow \begin{cases} 13x = -5x + 12y + 11 \\ 13y = -12x - 5y + 3 \end{cases} \Rightarrow \boxed{C\left(\frac{1}{2} ; -\frac{1}{6}\right)}$$

ψ è la ROTAZ. IN SENSO ORARIO di $\alpha = \arccos\left(-\frac{5}{13}\right)$ CON CENTRO.

4) IMM. RISP. A r:

$$\begin{cases} x' = ax + by + h \\ y' = bx - ay + k \end{cases}$$

$h = k = 0$

$(0;0) \mapsto (0;0)$

$(1;3) \mapsto (1;3)$

$$\begin{cases} 1 = a \cdot 1 + b \cdot 3 \\ 3 = b \cdot 1 - a \cdot 3 \end{cases} \Rightarrow \begin{cases} a = -4/5 \\ b = 3/5 \end{cases}$$

SIMMETRIA: $\begin{cases} x' = -4/5\, x + 3/5\, y \\ y' = 3/5\, x + 4/5\, y \end{cases}$

<u>GLISSOSIMMETRIA</u>: $\begin{cases} x'' = x' + h \\ y'' = y' + k \end{cases}$

$$\begin{cases} x'' = -\frac{4}{5}x + \frac{3}{5}y + h \\ y'' = \frac{3}{5}x + \frac{4}{5}y + k \end{cases}$$

$(5;0) \mapsto (-2;9)$, QUINDI TROVO h, k:

$$\begin{cases} -2 = -\frac{4}{5} \cdot 5 + \frac{3}{5} \cdot 0 + h \\ 9 = \frac{3}{5} \cdot 5 + \frac{4}{5} \cdot 0 + k \end{cases} \Rightarrow \begin{cases} h = 2 \\ k = 6 \end{cases}$$

IN DEFINITIVA:

$$\boxed{\begin{cases} x'' = -\frac{4}{5}x + \frac{3}{5}y + 2 \\ y'' = \frac{3}{5}x + \frac{4}{5}y + 6 \end{cases}}$$

N.B.
SI NOTI CHE
$\begin{pmatrix} 2 \\ 6 \end{pmatrix} // \begin{pmatrix} 1 \\ 3 \end{pmatrix} // r$.

5) assi di simmetria: $y=-\frac{1}{3}x$; $y=3x$.

$$dist(O;d_1)=\frac{|3\cdot 0-y-4\sqrt{5}|}{\sqrt{3^2+1^2}}=\frac{4\sqrt{5}}{\sqrt{10}}=2\sqrt{2}.$$

$$a=dist(O;V_1)=\sqrt{10}.$$

$$\frac{a^2}{c}=2\sqrt{2}\Rightarrow\frac{10}{c}=2\sqrt{2}\Rightarrow c=\frac{10}{2\sqrt{2}}=\frac{5}{\sqrt{2}}.$$

$$c^2=a^2+b^2\Rightarrow b^2=c^2-a^2=\frac{25}{2}-10=\frac{5}{2}\quad\left(b=\sqrt{\frac{5}{2}}\right).$$

$$\frac{x^2}{10}-\frac{y^2}{5/2}=1\longrightarrow x^2-10\cdot\frac{2}{5}\cdot y^2=10$$

QUINDI PARTIAMO DA $x^2-4y^2=10$.

$$\begin{cases}x'=\frac{3}{\sqrt{10}}x+\frac{1}{\sqrt{10}}y\\ y'=-\frac{1}{\sqrt{10}}x+\frac{3}{\sqrt{10}}y\end{cases}\rightarrow\begin{cases}x=\frac{3}{\sqrt{10}}x'-\frac{1}{\sqrt{10}}y'\\ y=\frac{1}{\sqrt{10}}x'+\frac{3}{\sqrt{10}}y'\end{cases}$$

$$\left(\frac{3}{\sqrt{10}}x'-\frac{y'}{\sqrt{10}}\right)^2-4\cdot\left(\frac{x'}{\sqrt{10}}+\frac{3}{\sqrt{10}}y'\right)^2-10=0\Rightarrow$$

$$\frac{1}{10}\cdot(3x'-y')^2-4\cdot\frac{1}{10}\cdot(x'+3y')^2-10=0\Rightarrow$$

$$9x'^2+y'^2-6x'y'-4\cdot(x'^2+9y'^2+6x'y')=100\rightarrow$$

$$5x'^2-30x'y'-35y'^2=100\Rightarrow$$

$$\boxed{x'^2-6x'y'-7y'^2=20}$$

FUOCHI:

$$\overrightarrow{OF_{1,2}}=\pm c\cdot\begin{pmatrix}3/\sqrt{10}\\-1/\sqrt{10}\end{pmatrix}=\pm\frac{5}{\sqrt{2}}\cdot\begin{pmatrix}3/\sqrt{10}\\-1/\sqrt{10}\end{pmatrix}\Rightarrow$$

$$\overrightarrow{OF_{1,2}}=\begin{pmatrix}\pm\frac{3}{2}\sqrt{5}\\ \mp\frac{\sqrt{5}}{2}\end{pmatrix}$$

eccentricità: $e = \frac{c}{a} = c \cdot \frac{1}{a} = \frac{5}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}} \Rightarrow$

$$e = \frac{\sqrt{5}}{2} \approx 1{,}118.$$

Asintoti: $y = \pm \frac{b}{a} x \Rightarrow y = \pm \frac{\sqrt{\frac{5}{2}}}{\sqrt{10}} x$

$$\Rightarrow y = \pm \frac{1}{2} x$$

$$\frac{1}{\sqrt{10}} x' + \frac{3}{\sqrt{10}} y' = \pm \frac{1}{2} \cdot \left(\frac{3}{\sqrt{10}} x' - \frac{1}{\sqrt{10}} y' \right) \Rightarrow$$

$$x' + 3y' = \pm \frac{1}{2} \cdot (3x' - y') \Rightarrow$$

$$2x' + 6y' = \pm (3x' - y') \Rightarrow$$

$$\boxed{y' = \frac{1}{7} x'} \quad \vee \quad \boxed{y' = -x'}$$

b) $\begin{cases} x' = ax + by + h \\ y' = cx + dy + k \end{cases}$ con $h = k = 0$ in quanto $(0;0)$ è punto fisso.

$$A(1;1) \in r \;\downarrow\; A'(2;2) \in r \Rightarrow \begin{cases} 2 = a + b \\ 2 = c + d \end{cases}$$

$$B(-2;1) \in s \;\downarrow\; B'(6;-3) \in s \Rightarrow \begin{cases} 6 = -2a + b \\ -3 = -2c + d \end{cases}$$

devo risolvere un sistema 4×4 nelle incognite a, b, c, d:

$$\begin{cases} a+b=2 \\ -2a+b=6 \\ c+d=2 \\ -2c+d=-3 \end{cases} \Rightarrow \begin{cases} a=-4/3 \\ b=10/3 \\ c=5/3 \\ d=1/3 \end{cases}$$

quindi

$$\varphi: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -4/3 & \frac{10}{3} \\ 5/3 & 1/3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$