1 $\quad (x+iy)^2+i(x+iy)+10i=k \quad (k\in\mathbb{R})$

$$x^2-y^2+2ixy+ix-y+10i-k=0$$

$$\begin{cases} x^2-y^2-y-k=0 \\ 2xy+x+10=0 \end{cases} \Rightarrow \begin{cases} 4-y^2-y-k=0 \\ 4y+12=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 4-9+3-k=0 \\ y=-3 \end{cases} \Rightarrow \begin{cases} k=-2 \\ y=-3 \end{cases}$$

$$z^2+iz+(2+10i)=0$$

UNA soluz. è $\boxed{z_1 = 2-3i}$

l'altra è t.c. $z_1 \cdot z_2 = \dfrac{2+10i}{1}$

$$z_2 = \frac{2+10i}{2-3i} = \frac{(2+10i)(2+3i)}{(2-3i)(2+3i)}$$

$$z_2 = \frac{-26+26i}{13} = \boxed{-2+2i}$$

2° METODO: $\quad z_1+z_2 = -\dfrac{i}{1} \Rightarrow z_2 = -i-z_1$

$$= -i-(2-3i)$$

$$= \boxed{-2+2i}.$$

(2)

$$(z-i)^4 = -\frac{32(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})} =$$

$$= \frac{-32(1-i\sqrt{3})}{1+3} = -8+8\sqrt{3}\,i$$

SOSTITUZIONE
$W = z - i$

$$W^4 = +16\cdot\left(-\frac{1}{2}+\frac{1}{2}\sqrt{3}\,i\right)$$

$$\rho = 16 \ ; \ \text{Arg.} = \frac{2}{3}\pi$$

$$W_k = \underbrace{\sqrt[4]{16}}_{2}\cdot\left[\cos\left(\frac{\frac{2}{3}\pi+2k\pi}{4}\right)+i\sin\left(\frac{\frac{2}{3}\pi+2k\pi}{4}\right)\right]$$

$$k=0 \longrightarrow 2\cdot\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=\sqrt{3}+i$$

$$k=1 \longrightarrow 2\cdot\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=-1+\sqrt{3}\,i$$

$$k=2 \longrightarrow 2\cdot\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)=-\sqrt{3}-i$$

$$k=3 \longrightarrow 2\cdot\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=1-\sqrt{3}\,i$$

$$z = W + i$$

$$\boxed{\begin{aligned} z_0 &= \sqrt{3}+2i \\ z_1 &= -1+(\sqrt{3}+1)i \\ z_2 &= -\sqrt{3} \\ z_3 &= 1+(1-\sqrt{3})i \end{aligned}}$$

(3) $$\left(\frac{x+\cancel{iy}+x-\cancel{iy}}{2}\right)^2+\left(\frac{\cancel{x}+iy-(\cancel{x}-iy)}{2i}\right)^2-2(x+iy)+2i=k$$

$$x^2+y^2-2x-2iy+2i-k=0$$

$$\begin{cases} x^2+y^2-2x-k=0 \\ 2y=2 \end{cases} \longrightarrow \begin{cases} x^2-2x+1-k=0 \\ y=1 \end{cases}$$

LE SOLUZIONI <u>ESISTONO</u> SE E SOLO SE

$$\Delta=4-4\cdot(1-k)\geq 0 \Rightarrow \boxed{k\geq 0}$$

CON $K=4 \rightarrow \quad x_{1,2}=\frac{2\pm\sqrt{16}}{2}= \begin{cases} 3 \\ -1 \end{cases}$

le soluzioni sono: $\boxed{z_1=3+i\ ,\ z_2=-1+i}$.

④ $z^2 = -i|z|^2$

$$(x+iy)^2 = -i\cdot(x^2+y^2)$$

$$x^2-y^2+2ixy = -ix^2-iy^2$$

$$\begin{cases} x^2-y^2=0 \\ x^2+y^2+2xy=0 \end{cases} \to \begin{cases} (x+y)(x-y)=0 \\ (x+y)^2=0 \end{cases}$$

LE SOLUZIONI SONO:

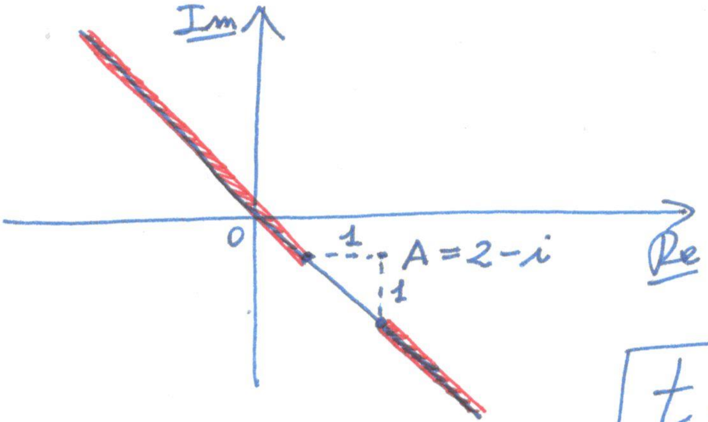
$$\begin{cases} x+y=0 \\ (x+y)^2=0 \end{cases} \vee \begin{cases} x-y=0 \\ (x+y)^2=0 \end{cases}$$

$$\downarrow$$

$$\underbrace{\begin{cases} x=t \\ y=-t \end{cases} \vee \begin{cases} x=0 \\ y=0 \end{cases}}$$

$$\begin{cases} x=t \\ y=-t \end{cases} \quad t\in\mathbb{R}$$

QUINDI: $z = x+iy = \boxed{t-ti}$



$$|z-2+i| \geq 1$$

$$|z-(2-i)| \geq 1$$

$$\boxed{t \leq 1 \vee t \geq 2}$$

(5)

$$z' = \bar{z}$$

$$z'' = -i\,(z' - 1 + 2i) + 1 - 2i$$

$$z''' = \overline{z''}$$

$$z''' = \overline{-i\,(\bar{z} - 1 + 2i) + 1 - 2i} =$$

$$= \overline{-i} \cdot (\bar{\bar{z}} - 1 - 2i) + 1 + 2i =$$

$$= i \cdot (z - 1 - 2i) + 1 + 2i$$

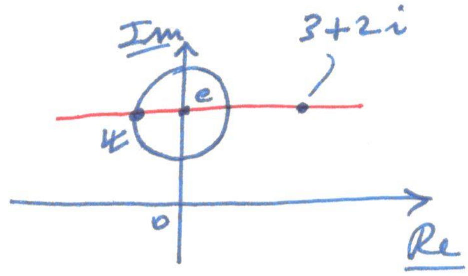
$$\boxed{z_{e^*} = 1 + 2i}$$

ROTAZ. di 90° IN SENSO ANTIORARIO ATTORNO AL PUNTO $e^*(1;2)$.

(6)

$$\frac{|z|}{|z-i|} = \sqrt{2} \Rightarrow |z| = \sqrt{2} \cdot |z-i|$$

$$\Rightarrow |z|^2 = 2 \cdot |z-i|^2$$



$$x^2 + y^2 = 2 \cdot [x^2 + (y-1)]^2$$

$$x^2 + y^2 = 2 \cdot (x^2 + y^2 - 2y + 1)$$

$x^2 + y^2 - 4y + 2 = 0$ circonf. di centro $c(0;2)$ e raggio $= \sqrt{2}$

PER DETERM. W E A :

$$\begin{cases} x^2 + y^2 - 4y + 2 = 0 \\ y = 2 \end{cases} \rightarrow \begin{cases} x^2 = 2 \\ y = 2 \end{cases} \rightarrow \begin{cases} x = \pm\sqrt{2} \\ y = 2 \end{cases}$$

$$\boxed{W = -\sqrt{2} + 2i}$$