SOLUZIONI III° CLASSICO – 23/12/15

1) $k\dfrac{Q_1}{(l/9)^2} = k\dfrac{Q_2}{\left(\frac{8}{9}l\right)^2} \Rightarrow k\cdot\dfrac{Q_1}{l^2}\cdot 81 = k\cdot\dfrac{Q_2}{l^2}\cdot\dfrac{81}{64} \Rightarrow \boxed{\dfrac{Q_1}{Q_2} = \dfrac{1}{64}}$

2) $Q = 4\pi r^2\cdot\sigma \Rightarrow Q = 4\pi\cdot(2{,}7\cdot 10^{-2}\,m)^2\cdot 3\cdot 10^{-8}\,C/m^2 = \boxed{2{,}7\cdot 10^{-10}\,C}$

3) $k\cdot\dfrac{q_1\cdot(Q-q_1)}{r^2} = F \Rightarrow kQq_1 - kq_1^2 - r^2F = 0 \Rightarrow$

$\Rightarrow q_1 = \dfrac{kQ \pm\sqrt{k^2Q^2 - 4kr^2F}}{2k} = \begin{cases}\boxed{4\cdot 10^{-6}\,C}\\ \boxed{2\cdot 10^{-6}\,C}\end{cases}$

4) $M|a| = |q|\cdot E \Rightarrow |a| = \dfrac{|q|\cdot E}{M}$; $s = 0 + v_i\cdot t - \dfrac{1}{2}\,\dfrac{|q|\cdot E}{M}\cdot t^2$

$s_f = 0 + 6\cdot 10^4\cdot(0{,}7\cdot 10^{-9}) - \dfrac{1}{2}\cdot\dfrac{(1{,}6\cdot 10^{-19})\cdot 40}{9{,}1\cdot 10^{-31}}\cdot(0{,}7\cdot 10^{-9})^2 \cong \boxed{4{,}0\cdot 10^{-5}\,m}$

$v_f = 6\cdot 10^4 - \dfrac{|q|\cdot E}{M}\cdot t \Rightarrow v_f = 6\cdot 10^4 - \dfrac{(1{,}6\cdot 10^{-19})\cdot 40}{9{,}1\cdot 10^{-31}}\cdot(0{,}7\cdot 10^{-9}) =$
$\cong \boxed{5{,}5\cdot 10^4\,m/s\,(>0)}$

5) $E = \dfrac{\lambda}{2\pi\varepsilon_0\cdot r} \Rightarrow \lambda = E\cdot 2\pi\varepsilon_0\cdot r = \boxed{1{,}27\cdot 10^{-5}\,C/m}$

6) $\sigma < 0$; $F = Ma \Rightarrow |q|\cdot E = M\cdot|a| \Rightarrow |q|\cdot\dfrac{|\sigma|}{2\varepsilon_0} = M\cdot|a| \Rightarrow$

$\Rightarrow |a| = \dfrac{|q|\cdot|\sigma|}{2\varepsilon_0\cdot M}$; $v_f = 0 + |a|\cdot t \Rightarrow v_f = \dfrac{|q|\cdot|\sigma|}{2\varepsilon_0\cdot M}\cdot t \Rightarrow$

$\Rightarrow |\sigma| = \dfrac{2\varepsilon_0 M\cdot v_f}{|q|\cdot t} = \dfrac{2\cdot(8{,}85\cdot 10^{-12})\cdot(5\cdot 10^{-1})\cdot 3{,}4}{(56\cdot 10^{-3})\cdot(5\cdot 60)} \cong 1{,}8\cdot 10^{-12}\,C/m^2$

QUINDI, essendo $\sigma < 0$, RISULTA: $\boxed{\sigma = -1{,}8\cdot 10^{-12}\,C/m^2}$

7) a) $E\cdot(4\pi r^2) = \dfrac{Q_{INT}}{\varepsilon_0} \Rightarrow E\cdot(4\pi r^2) = \dfrac{\frac{4}{3}\pi r^3\cdot\rho}{\varepsilon_0} \rightarrow$

$E = \dfrac{4}{3}\pi r^3\rho\cdot\dfrac{1}{4\pi r^2}\cdot\dfrac{1}{\varepsilon_0} \Rightarrow E = \dfrac{\rho\cdot r}{3\cdot\varepsilon_0} = \dfrac{4{,}6\cdot 10^{-6}\cdot(3\cdot 10^{-2})}{3\cdot 8{,}85\cdot 10^{-12}}$

$\Rightarrow \boxed{E \cong 5{,}2\cdot 10^3\,N/C}$

b) $E\cdot(4\pi r^2) = \dfrac{Q_{INT}}{\varepsilon_0} \rightarrow E\cdot(4\pi r^2) = \dfrac{\frac{4}{3}\pi R^3\cdot\rho}{\varepsilon_0} \Rightarrow$

$\Rightarrow E = \dfrac{4}{3}\pi R^3\cdot\rho\cdot\dfrac{1}{4\pi r^2\varepsilon_0} \rightarrow E = \dfrac{\rho\cdot R^3}{3r^2\varepsilon_0} = \dfrac{(4{,}6\cdot 10^{-6})\cdot(7\cdot 10^{-2})^3}{3\cdot(8\cdot 10^{-2})^2\cdot(8{,}85\cdot 10^{-12})}$

$\rightarrow \boxed{E \cong 9{,}3 \cdot 10^{3}\ N/C}$; il campo max si ottiene con la formula

$E = \dfrac{\rho \cdot R^3}{3 r^2 \cdot \varepsilon_0}$ sostituendo $r = R \Rightarrow E_{MAX} = \dfrac{\rho \cdot R^3}{3 \cdot R^2 \cdot \varepsilon_0} = \dfrac{\rho \cdot R}{3 \cdot \varepsilon_0} \Rightarrow$

$$E_{MAX} = \frac{(4{,}6 \cdot 10^{-6}) \cdot (7 \cdot 10^{-2})}{3 \cdot (8{,}85 \cdot 10^{-12})} \cong 1{,}2 \cdot 10^4\ N/C.$$

8) $F = k \dfrac{q_1 q_2}{r^2} \Rightarrow r = \sqrt{\dfrac{k\, q_1 q_2}{F}} \cong \boxed{5\ m}$.

9) $E = \dfrac{Q \cdot z}{4\pi\varepsilon_0 \cdot (z^2+R^2)^{3/2}} \rightarrow (z^2+R^2)^{3/2} = \dfrac{Q \cdot z}{4\pi\varepsilon_0 \cdot E} \Rightarrow$

$\left[(z^2+R^2)^{3/2}\right]^{2/3} = \left[\dfrac{Q \cdot z}{4\pi\varepsilon_0 \cdot E}\right]^{2/3} \rightarrow z^2+R^2 = \left(\dfrac{Q \cdot z}{4\pi\varepsilon_0 \cdot E}\right)^{2/3} \Rightarrow$

$R^2 = \left(\dfrac{Q \cdot z}{4\pi\varepsilon_0 \cdot E}\right)^{2/3} - z^2 \Rightarrow R = \sqrt{\left(\dfrac{Q \cdot z}{4\pi\varepsilon_0 \cdot E}\right)^{2/3} - z^2} \cong \boxed{2 \cdot 10^{-2}\ m}$

10) $M \cdot a = F \Rightarrow M \cdot \dfrac{v^2}{r} = k\, \dfrac{q_1 \cdot q_2}{r^2} \Rightarrow v = \sqrt{\dfrac{k \cdot q_1 \cdot q_2}{M \cdot r}} \Rightarrow$

$v = \dfrac{9 \cdot 10^9 \cdot (1{,}6 \cdot 10^{-19})^2}{(9{,}1 \cdot 10^{-31}) \cdot (5{,}3 \cdot 10^{-11})} \Rightarrow \boxed{v \cong 2{,}2 \cdot 10^6\ m/s}$

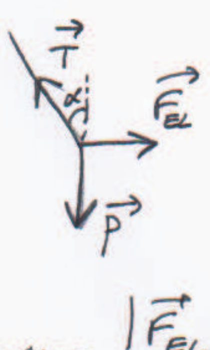
11) $E_1 = \dfrac{1}{4} \cdot E_0 \Rightarrow \dfrac{\sigma}{2\varepsilon_0} \cdot \left(1 - \dfrac{z}{\sqrt{z^2+R^2}}\right) = \dfrac{1}{4} \cdot \dfrac{\sigma}{2\varepsilon_0} \cdot \left(1 - \dfrac{0}{\sqrt{0^2+R^2}}\right)$

$\Rightarrow 1 - \dfrac{z}{\sqrt{z^2+R^2}} = \dfrac{1}{4} \Rightarrow \dfrac{z}{\sqrt{z^2+R^2}} = \dfrac{3}{4} \Rightarrow \dfrac{z^2}{z^2+R^2} = \dfrac{9}{16} \Rightarrow$

$z^2 \cdot 16 = 9z^2 + 9R^2 \Rightarrow 7z^2 = 9R^2 \Rightarrow z^2 = \dfrac{9}{7} R^2 \Rightarrow z = \dfrac{3}{\sqrt{7}} R \Rightarrow$

$z = \dfrac{3}{\sqrt{7}} \cdot (0{,}8) \Rightarrow \boxed{z \cong 0{,}91\ m}$.

12)



$\vec{T} + \vec{F}_{EL} + \vec{P} = \vec{0} \Rightarrow \vec{T} = -(\vec{F}_{EL} + \vec{P})$

$|\vec{T}| = \sqrt{|\vec{F}_{EL}|^2 + |\vec{P}|^2} = \sqrt{(0{,}216N)^2 + (0{,}784N)^2} \Rightarrow$

$\boxed{|\vec{T}| = 0{,}813N}$

$\tan\alpha = \dfrac{|\vec{F}_{EL}|}{|\vec{P}|} \Rightarrow \alpha = \arctan\left(\dfrac{0{,}216N}{0{,}784N}\right) = \boxed{15{,}4^\circ}$

13) $\begin{cases} x = 0 + v_0 \cdot t \\ y = 0 + 0 \cdot t + \frac{1}{2} \cdot \frac{qE}{M} \cdot t^2 \end{cases}$ $\Rightarrow$ LEGGE ORARIA DELL'ELETTRONE

SE VOGLIAMO LA TRAIETTORIA ELIMINIAMO t:

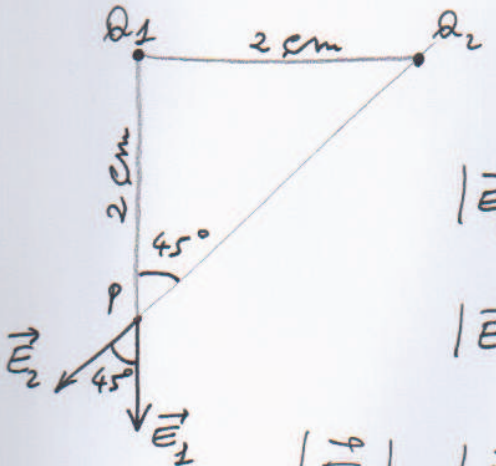
$$\begin{cases} t = \frac{x}{v_0} \\ y = \frac{1}{2} \frac{qE}{M} \left(\frac{x}{v_0}\right)^2 \end{cases} \Rightarrow \boxed{y = \frac{qE}{2Mv_0^2} x^2}$$

Sostituisco $x = 2 \cdot 10^{-2}\,m$ e $y = 1{,}5 \cdot 10^{-3}\,m$:

$$E = \frac{2Mv_0^2 \cdot y}{q \cdot x^2} \Rightarrow E = \frac{2 \cdot (9{,}1 \cdot 10^{-31}) \cdot (7 \cdot 10^6)^2 \cdot (1{,}5 \cdot 10^{-3})}{(1{,}6 \cdot 10^{-19}) \cdot (2 \cdot 10^{-2})^2} \Rightarrow$$

$\boxed{E \cong 2{,}1 \cdot 10^3\,N/C}$.

14)



$$|\vec{E}_1| = k \cdot \frac{Q_1}{r_1^2} = 6{,}75 \cdot 10^5\,N/C$$

$$|\vec{E}_2| = k \cdot \frac{Q_2}{r_2^2} = 6{,}75 \cdot 10^5\,N/C$$

$|\vec{E}_1| = |\vec{E}_2| = 6{,}75 \cdot 10^5\,N/C$; PONGO $E = |\vec{E}_1| = |\vec{E}_2|$

$$\vec{E}_1 = \begin{pmatrix} 0 \\ -E \end{pmatrix};\ \vec{E}_2 = \begin{pmatrix} -E/\sqrt{2} \\ -E/\sqrt{2} \end{pmatrix} \rightarrow \vec{E}_1 + \vec{E}_2 = \begin{pmatrix} -E/\sqrt{2} \\ -E - E/\sqrt{2} \end{pmatrix}$$

$$|\vec{E}_1 + \vec{E}_2| = \sqrt{\left(-\frac{E}{\sqrt{2}}\right)^2 + \left(-E - \frac{E}{\sqrt{2}}\right)^2} \cong \boxed{1{,}25 \cdot 10^6\,N/C}$$

OPPURE:

$$|\vec{E}_1 + \vec{E}_2| = 2 \cdot E \cdot \cos\left(\frac{45°}{2}\right) = 2 \cdot (6{,}75 \cdot 10^5) \cdot \cos(22{,}5°) \rightarrow$$

$$\Rightarrow |\vec{E}_1 + \vec{E}_2| \cong 1{,}25 \cdot 10^6\,N/C.$$